PROOF OF FORMULA 4.331.2

$$\int_{1}^{\infty} e^{-\mu x} \ln x \, dx = -\frac{1}{\mu} \text{Ei}(-\mu)$$

Integrate by parts to obtain

$$\int_1^\infty e^{-\mu x} \ln x \, dx = \frac{1}{\mu} \int_1^\infty \frac{e^{-\mu x}}{x} \, dx.$$

 $\int_1^\infty e^{-\mu x} \ln x \, dx = \frac{1}{\mu} \int_1^\infty \frac{e^{-\mu x}}{x} \, dx.$ The result now follows via the change $s=\mu x$ and the definition of the exponential integral

$$\mathrm{Ei}(x) = -\int_{x}^{\infty} \frac{e^{-t}}{t} dt.$$