PROOF OF FORMULA 4.335.3

$$\int_0^\infty e^{-\mu x} \ln^3 x \, dx = -\frac{1}{2\mu} \left[2(\gamma + \ln \mu)^3 + \pi^2 (\gamma + \ln \mu) - \psi''(1) \right]$$

Start with

$$\int_0^\infty x^a e^{-\mu x} \, dx = \mu^{-(a+1)} \Gamma(a+1)$$

and differentiate three times with respect to a. Then put $a = 0$ to obtain

$$\int_0^\infty e^{-\mu x} \ln^3 x \, dx = -\frac{1}{\mu} \left[-(\psi(1) - \ln \mu)^3 - 3\psi'(1)(\psi(1) - \ln \mu) - \psi''(1) \right].$$

The values $\psi(1) = -\gamma$ and $\psi'(1) = \pi^2/6$ give the result.