

## Bifurcation of periodic solutions in nonlinear evolution problems with periodic forcing

This lecture will address the bifurcation of  $2\pi$ -periodic solutions in nonautonomous semilinear PDEs of evolution type with  $2\pi$ -periodic forcing. For expository purposes, we shall center the discussion on the “model” problem

$$\begin{cases} u_t = \Delta u + f(\lambda, t, u), \\ u = 0 \text{ on } \mathbb{R} \times \partial\Omega, \end{cases}$$

where  $\Omega$  is a bounded open subset of  $\mathbb{R}^N$ ,  $\lambda$  is a real parameter and  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  is a given function such that  $f(\lambda, t, \xi)$  is  $2\pi$ -periodic in  $t$  for all  $\lambda, \xi \in \mathbb{R}$  and  $f(\lambda, t, 0) = 0$ . Thus,  $u = 0$  is a  $2\pi$ -periodic solution for all  $\lambda$  and bifurcation will be investigated from this trivial branch.

The more traditional way to proceed is to reduce the question to the bifurcation of fixed points of the Poincaré map and next to make the problem finite-dimensional by a center manifold theorem. The actual dimension depends upon how many Floquet multipliers of the linearization lie on or outside the unit circle. Thus, information about the multipliers is needed and the finite dimensional problem may still be nontrivial if the dimension is “large”.

We shall describe a different approach in which bifurcation is studied directly in Sobolev-like spaces of  $2\pi$ -periodic functions. This approach relies on recent results about the Fredholm and spectral properties of linear evolution operators in such spaces. In such a setting, the rather well developed theory of local or global bifurcation for Fredholm operators of index 0 becomes available. The most notable features are that

(i) The method does not require or imply the well-posedness of the forward or backward Cauchy problem. As a result, not only the Floquet multipliers are not involved in the discussion, but they need not even exist. The latter is not an issue with the model problem, but a nonparabolic example will be given when this advantage is significant.

(ii) The bifurcation/nonbifurcation points are characterized by an associated stationary problem.

(iii) The functional setting allows for time-discontinuities in the forcing term  $f$ .

The lecture will identify the key steps and important concepts without going into minute technical arguments.