MATH 115 – FINAL EXAM ANSWER SHEET

Thursday, May 5, 2005, 8:00 am – 12:00 pm

Name: ______________________________ Instructor: ________________________

1. **DO NOT SEPARATE** answer sheet from rest of test.
2. Work and CIRCLE the one answer to each problem INSIDE this test.
3. Circle your answer a SECOND TIME on this page.
4. Blank answers are considered INCORRECT; no penalty for wrong answers.
5. Do all problems, each carries equal weight.
6. Turn in ENTIRE TEST, showing ALL relevant work leading to your answer in the space provided.

QUESTION	ANSWER					QUESTION	ANSWER				
1.	a	b	c	d	e	16.	a	b	c	d	e
2.	a	b	c	d	e	17.	a	b	c	d	e
3.	a	b	c	d	e	18.	a	b	c	d	e
4.	a	b	c	d	e	19.	a	b	c	d	e
5.	a	b	c	d	e	20.	a	b	c	d	e
6.	a	b	c	d	e	21.	a	b	c	d	e
7.	a	b	c	d	e	22.	a	b	c	d	e
8.	a	b	c	d	e	23.	a	b	c	d	e
9.	a	b	c	d	e	24.	a	b	c	d	e
10.	a	b	c	d	e	25.	a	b	c	d	e
11.	a	b	c	d	e	26.	a	b	c	d	e
12.	a	b	c	d	e	27.	a	b	c	d	e
13.	a	b	c	d	e	28.	a	b	c	d	e
14.	a	b	c	d	e	29.	a	b	c	d	e
15.	a	b	c	d	e	30.	a	b	c	d	e

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QUESTION	ANSWER					QUESTION	ANSWER				
31.	a	b	c	d	e	46.	a	b	c	d	e
32.	a	b	c	d	e	47.	a	b	c	d	e
33.	a	b	c	d	e	48.	a	b	c	d	e
34.	a	b	c	d	e	49.	a	b	c	d	e
35.	a	b	c	d	e	50.	a	b	c	d	e
36.	a	b	c	d	e	51.	a	b	c	d	e
37.	a	b	c	d	e	52.	a	b	c	d	e
38.	a	b	c	d	e	53.	a	b	c	d	e
39.	a	b	c	d	e	54.	a	b	c	d	e
40.	a	b	c	d	e	55.	a	b	c	d	e
41.	a	b	c	d	e	56.	a	b	c	d	e
42.	a	b	c	d	e	57.	a	b	c	d	e
43.	a	b	c	d	e	58.	a	b	c	d	e
44.	a	b	c	d	e	59.	a	b	c	d	e
45.	a	b	c	d	e	60.	a	b	c	d	e

MATH 115 - FINAL EXAM

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PRINT: Name ______________________________ Section ____________

1. Find the domain of $f(x) = \frac{1}{\sqrt{(x-1)(2x+3)}}$

   (a) $[\frac{3}{2},\ 1]$
   (b) $[-\frac{3}{2},\ 1)$
   (c) $(-\infty,\ -\frac{3}{2}) \cup (1,\ \infty)$
   (d) $(-\infty,\ -\frac{3}{2}] \cup [1,\ \infty)$
   (e) none of the above

2. Find an equation of the line that is parallel to $y = 15x - 7$ and passes through $(-1,\ 8)$.

   (a) $y = 8x - 1$
   (b) $y = 15x - 7$
   (c) $y = 15x + 7$
   (d) $y = 15x + 23$
   (e) none of the above

3. An equation of the line through $(2,\ 3)$ and perpendicular to $x + 4y = 5$ is

   (a) $y = 4x - 5$
   (b) $y = -4x + 10$
   (c) $y = -\frac{1}{4}x + \frac{11}{4}$
   (d) $y = \frac{1}{4}x + \frac{5}{4}$
   (e) none of the above

4. If $g(x) = x^2$ and $f(g(x)) = \frac{1}{x^2+1}$ what is f?

   (a) $f(x) = \frac{1}{x^2+1}$
   (b) $f(x) = \frac{1}{x+1}$
   (c) $f(x) = \frac{1}{x}$
   (d) $f(x) = \frac{1}{x^2}$
   (e) none of the above

5. Suppose that $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, and $h(x) = 1 - 2x + x^2$ what is $g \circ h \circ f$?

(a) $\frac{1}{1+(x^2+1)^2}$

(b) $\frac{1}{(1-2x+x^2)^2}$

(c) $(\frac{1}{x})^2 + 1$

(d) $\frac{1}{x^4}$

(e) none of the above

6. Which function is even

(a) $f(x) = 1 - 2x$

(b) $f(x) = -15x + 3$

(c) $f(x) = x^3 + x^2$

(d) $f(x) = x^{20} + x^{40}$

(e) none of the above

7. Find the inverse of the function $f(x) = \frac{x-1}{x+1}$

(a) $f^{-1}(x) = \frac{x-2}{x+2}$

(b) $f^{-1}(x) = \frac{1}{x+1}$

(c) $f^{-1}(x) = \frac{x}{x-1}$

(d) $f^{-1}(x) = \frac{-(x+1)}{(x-1)}$

(e) none of the above

8. $\displaystyle\lim_{x\to\infty} \frac{1+2x}{x-1} = ?$

(a) 1

(b) -1

(c) -2

(d) 2

(e) none of the above

9. $\lim_{x\to 7^+} \sqrt{2x}\dfrac{(x-7)}{|x-7|} = ?$

(a) ∞
(b) $-\infty$
(c) $\sqrt{14}$
(d) $\sqrt{15}$
(e) none of the above

10. $\lim_{x\to -9^+} (x+3)\dfrac{|x+9|}{x+9} = ?$

(a) -6
(b) 6
(c) 12
(d) 9
(e) none of the above

11. $\lim_{x\to -5} \left((x-1)^2 - x^2\right) = ?$

(a) -50
(b) 30
(c) 11
(d) -40
(e) none of the above

12. $\lim_{h\to 0} \dfrac{\sqrt{9+h}-3}{h} = ?$

(a) $\frac{3}{9}$
(b) $\frac{9}{3}$
(c) ∞
(d) $\frac{1}{6}$
(e) none of the above

13. Simplify: $\dfrac{x^{n+1}y^{2n}}{(\frac{x}{y})^n}$

(a) $\frac{x^n}{y^n}$

(b) $\frac{x^{n+1}}{y^n}$

(c) $\frac{y^{3n}}{x}$

(d) xy^{3n}

(e) none of the above

14. $f(x) = \begin{cases} 3x+2 & if \quad x < -1 \\ 5+4x^3 & if \quad x \geq -1 \end{cases}$

Determine whether f is left continuous at $c = -1$ and whether f is right continuous at $c = -1$.

(a) Right continuous

(b) Left continuous

(c) Both left and right continuous

(d) Neither left nor right continuous

(e) none of the above

15. The vertex of $y = f(x) = -2x^2 + 4x + 11$ is

(a) $(-1, 5)$

(b) $(1, 13)$

(c) $(2, 11)$

(d) 11

(e) none of the above

16. $f(x)$ is undefined at $x = 1$. Define $f(1)$ so that $f(x)$ is continuous at $x = 1$, where $f(x) = \frac{x^2-2x+1}{x^2-6x+5}$.

(a) Define $f(1) = 2$

(b) Define $f(1) = 3$

(c) Define $f(1) = 1$

(d) Define $f(1) = 0$

(e) none of the above

17. Find the global max and min values of $f(x) = 2x^3 - 15x^2 + 24x + 20$ on $[0, 6]$.

(a) min at $(4, 4)$ max at $(6, 56)$
(b) min at $(4, 4)$ no max
(c) min at $(4, 0)$ no max
(d) min at $(4, 4)$ max at $(1, 31)$
(e) none of the above

18. Find real numbers a and b for which the sum of squares of a and b is as small as possible. And $a + b = 35$.

(a) $a = 35,\ b = 0$
(b) $a = 0,\ b = 35$
(c) $a = 5,\ b = 30$
(d) $a = \frac{35}{2},\ b = \frac{35}{2}$
(e) none of the above

19. Complete the square: $y = \frac{3}{2}x^2 + 2x - 5$
State the vertex.

(a) $y = \frac{3}{2}(x - \frac{2}{3})^2 - \frac{20}{3}$ $\quad(\frac{2}{3}, \frac{20}{3})$
(b) $y = \frac{3}{2}(x + \frac{2}{3})^2 + \frac{20}{3}$ $\quad(-\frac{20}{3}, \frac{2}{3})$
(c) $y = \frac{3}{2}(x + \frac{2}{3})^2 - \frac{17}{3}$ $\quad(-\frac{2}{3}, -\frac{17}{3})$
(d) $y = \frac{3}{2}(x - \frac{2}{3})^2 + \frac{17}{3}$ $\quad(\frac{2}{3}, \frac{17}{3})$
(e) none of the above

In the next three problems, the height of a ball in feet, t seconds after it is thrown in the air (on an asteroid where gravity is weaker) is given by $h(t) = -t^2 + 6t + 7$.

20. The maximum height reached by the ball is (in ft)

(a) 13 (b) 3 (c) 16 (d) 20 (e) none of the above

21. When does the ball strike the ground (in seconds)?

(a) 1 (b) 6 (c) 7 (d) 5 (e) none of the above

22. With what velocity does the ball strike the ground (in ft/sec)?

(a) 4 (b) -4 (c) 0 (d) -8 (e) none of the above

23. An equation of the line tangent to the graph of $f(x) = x^2$ at $x = 1$ is

(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = 2x - 1$
(d) $y = x - 1$
(e) none of the above

24. If $f(x) = \pi^2 + 1$ then $f'(x)$ is

(a) 2π
(b) $2x$
(c) 0
(d) 1
(e) none of the above

25. The function $f(x) = |x|$ is differentiable

(a) everywhere
(b) only for $x > 0$
(c) only for $x < 0$
(d) only where $x = 0$
(e) none of the above

26. The function $f(x) = x^2 + bx + c$

(a) has an absolute maximum
(b) has an absolute minimum
(c) has both an absolute maximum and an absolute minimum
(d) has a local maximum
(e) none of the above

27. Using the Intermediate Value Theorem, find an appropriate (small) interval containing a solution of $x^4 + x^3 = 20$

(a) (-1, 1)
(b) (0, 1)
(c) (1, 2)
(d) (-1, 0)
(e) none of the above

28. If $f(x) = \sqrt{x^2+1}$ then f' is

(a) $\frac{1}{2}(x^2+1)^{-1/2}$
(b) $2x$
(c) $\frac{1}{2}(x^2+1)^{-1/2} \cdot x$
(d) $\frac{1}{\sqrt{x^2+1}}$
(e) none of the above

29. If $f(x)$ is differentiable then

(a) it is continuous
(b) it passes the horizontal line test
(c) it is a polynomial
(d) it has an absolute maximum
(e) none of the above

30. If $f(x) = -\frac{1}{x}$ then $f''(x)$ is

(a) $\frac{1}{x^2}$
(b) $-\frac{1}{x^2}$
(c) $\frac{1}{x^4}$
(d) $\frac{2}{x^3}$
(e) none of the above

31. The critical points of $f(x) = \frac{x^3}{3} + x$ are

(a) 1 and -1
(b) 1
(c) -1
(d) 0 and $-\frac{1}{3}$
(e) none of the above

32. The function $f(x) = x^3 - 3x + 2$ has

(a) local maximum at 1
(b) local minimum at -1
(c) local maximum at -1 and local minimum at 1
(d) local minimum at -1 and local maximum at $+1$
(e) none of the above

33. The function $f(x) = 4x^3 + 9x^2 - 12x + 3$ increases on the set

(a) $(-\infty,\ 0)$
(b) $(0,\ \infty)$
(c) $(-\infty,\ -2) \cup (-2,\ \frac{1}{2})$
(d) $(-\infty,\ -2) \cup (\frac{1}{2},\ \infty)$
(e) none of the above

34. The function $f(x) = 3x^4 - 4x^3$ is concave up on the set

(a) $(0,\ \frac{2}{3})$
(b) $(-\infty,\ 0) \cup (\frac{2}{3},\ \infty)$
(c) $(0,\ 1)$
(d) $(-\infty,\ \frac{2}{3})$
(e) none of the above

35. The function $f(x) = x^4 - 6x^2 + 8x + 10$ has

(a) an inflection point at $x = 0$

(b) an inflection point only at $x = -1$

(c) an inflection point only at $x = 1$

(d) no inflection points

(e) none of the above

36. If $f(x) = \frac{9x^7}{x^2+1}$ then $f'(x)$ is

(a) $\frac{45x^8+63x^6}{(x^2+1)}$

(b) $\frac{45x^6+63x^8}{(x^2+1)^2}$

(c) $45x^8 + 63x^6$

(d) $\frac{45x^8+63x^6}{(x^2+1)^2}$

(e) none of the above

37. The absolute maximum of $f(x) = x - x^3$ on $[\,0,\ 1]$

(a) does not exist

(b) is $\frac{2}{9}\sqrt{3}$

(c) is $\frac{1}{3}\sqrt{3}$

(d) is 3

(e) none of the above

38. Among all rectangles of perimeter 2000 feet, one has maximum area.

(a) Its length is 1000 ft and width is 500 ft.

(b) Its length is 600 ft and width is 400 ft.

(c) Its length and width are the same.

(d) Its length is 750 ft and width is 250 ft.

(e) none of the above

39. If $f(x) = x\sqrt{x^4+1}$ then $f'(x)$ is

(a) $\frac{1}{2}\sqrt{x^4+1} \cdot 4x^3$
(b) $2x^4\sqrt{x^4+1}$
(c) $\sqrt{x^4+1}(2x^4+1)$
(d) $\sqrt{x^4+1}$
(e) none of the above

40. If $f'(c) = 0$ then

(a) $f(c) = 0$
(b) f has a local maximum at c
(c) $f''(c)$ is not zero
(d) the line tangent to the graph of f at c is vertical
(e) none of the above

41. If $f(0) = 0 = f(1)$ and f is differentiable on $(0,\ 1)$ then

(a) $f(x) = 0$ for all $x \in (0,\ 1)$
(b) $f'(x) = 0$ for all $x \in (0,\ 1)$
(c) $f'(x) = 0$ for some $x \in (0,\ 1)$
(d) $f(x) = 0$ for some $x \in (0,\ 1)$
(e) none of the above

42. If $f''(x) = 0$ for all $x \in \mathbb{R}$ then

(a) the graph of f is a parabola
(b) the graph of f is a cubic polynomial
(c) the graph of f is a line
(d) $f(x) = 0$ for all $x \in \mathbb{R}$
(e) none of the above

43. Find all asymptotes of $y = f(x) = \frac{x^2-2x-8}{x^2-x-6}$

(a) $x = 2,\ x = 3$

(b) $x = 2,\ x = 3,\ y = 1$

(c) $x = 3,\ y = 1$

(d) $x = 2,\ y = 1$

(e) none of the above

44. The derivative of $\frac{x^2-8x+6}{x^2-5x-7}$ is

(a) $\frac{2x-8}{2x-5}$

(b) $\frac{4x^3-39x^2+72x+26}{(x^2-5x-7)^2}$

(c) $\frac{3x^2-26x+86}{(x^2-5x-7)^2}$

(d) $\frac{3x^2+54x+86}{(x^2-5x-7)^2}$

(e) none of the above

45. What is the derivative of $f(x) = \sqrt{x + \sqrt{x}}$?

(a) $\frac{1}{\left[2\sqrt{x+\sqrt{x}}\right]}$

(b) $(1/2)(x + \sqrt{x})^{-1/2}$

(c) $(1 + \frac{1}{\sqrt{x}})\sqrt{x + \sqrt{x}}$

(d) $\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$

(e) none of the above

46. Find the tangent line to the curve $x^3 - xy + y^3 = 1$ at the point $(1,\ 1)$.

(a) $y = 3x - 2$

(b) $y = x$

(c) $y = x + 1$

(d) $y = -x + 2$

(e) none of the above

47. A point $P(x, y)$ moves along the curve $y^3 = 2(x+1)$ such that always $\frac{dx}{dt} = 12$. At that instant of time when $x = 3$, find $\frac{dy}{dt}$

(a) 2
(b) 4
(c) $\sqrt[3]{4}$
(d) $\frac{1}{4}$
(e) none of the above

48. Find the equation of the tangent line to $y^3 + y^2 = x^3 + x$ at $(1, 1)$

(a) $y = \frac{2x}{3} - \frac{1}{3}$
(b) $y = \frac{3x}{2} + \frac{1}{3}$
(c) $3y + 2y = 1$
(d) $y = \frac{4}{5}x + \frac{1}{5}$
(e) none of the above

49. For $y = f(x) = \frac{x^3+16}{x}$, find the x value (values) of all critical points for $0 < x < \infty$ only.

(a) $x = 4$
(b) $x = 2$
(c) $x = 2,\ 4$
(d) $x = 3\sqrt{2}$
(e) none of the above

50. The x-value of the minimum of $f(x) = x^{3/2}(3x - 5)$, occurs at

(a) $x = -\infty$
(b) $x = 0$
(c) $x = 1$
(d) $x = \frac{19}{17}$
(e) none of the above

51. For $f(x) = x - \frac{1}{x}, [1,\ 3]$ find all of the points $x = c$ guaranteed by the Mean Value Theorem.

(a) $\pm\sqrt{2}$

(b) $\frac{4}{3}$

(c) $\sqrt{3}$

(d) $\frac{1}{3}$

(e) none of the above

52. Find $\displaystyle\lim_{x \to -\infty} \frac{15x^3 + 2x^2 + 90}{3x^2 - x + 14}$

(a) $-\infty$

(b) ∞

(c) 5

(d) 15

(e) none of the above

53. Find all asymptotes of $y = f(x) = \frac{x^2+3x+9}{x+2}$

(a) $x = -2$

(b) $y = x + 1,\ x = -2$

(c) ∞

(d) $y = 2x + 1$

(e) none of the above

54. A spherical ballon is inflated at the rate of $16ft^3/minute$. How fast is its surface area increasing when the radius R is $2ft$? ($V = 4\pi R^3/3,\ A = 4\pi R^2$)

(a) 8

(b) 16

(c) $\frac{1}{\pi}$

(d) $\frac{2}{\pi}$

(e) none of the above

55. Find the x and y coordinates of the point on the line $x + 2y = 5$ closest to the origin.

(a) (1, 2)

(b) (2, 1)

(c) $(4, \frac{1}{2})$

(d) $(\frac{1}{2}, \frac{9}{4})$

(e) none of the above

56. A square based open top container is to be manufactured from a $6ft$ x $6ft$ square by cutting out a small square with side x feet from each corner, and then folding up the sides. What should x be to maximize the volume?

(a) 4

(b) 3

(c) 2

(d) 1

(e) none of the above

57. A rectangular container with a square base but open top is to contain $32in^3$. Find its dimensions, if the surface area of the container is to be a minimum. If x is the length of the side of the base, and y is the height, then x and y are:

(a) $x = 6, \quad y = 4$

(b) $x = 4, \quad y = 4$

(c) $x = 4, \quad y = 2$

(d) $x = 16, \quad y = 2$

(e) none of the above

58. Water is poured into a cylindrical container of radius $2ft$ at the rate of $8ft^3/sec$. How fast is the water level rising?

(a) $\frac{2}{\pi}$

(b) 4π

(c) $\frac{8}{\pi}$

(d) 4

(e) none of the above

59. An inverted cone of height 4 feet, radius 2 feet is being filled with water at the rate of $3ft^3/minute$. At that instant of time when the water level is 2 feet deep, how quickly is it rising?

(a) $\frac{4\pi}{3}$

(b) $\frac{8\pi}{9}$

(c) $\frac{3}{4\pi}$

(d) $\frac{3}{\pi}$

(e) none of the above

60. A $5ft$ ladder is leaning against a wall. The foot of the ladder is dragged away from the wall at the rate of $8ft/sec$. When the top of the ladder is $4ft$ above ground, how quickly is the top of the ladder descending?

(a) -6

(b) -4

(c) -8

(d) $-\frac{4}{\sqrt{5}}$

(e) none of the above