ANSWER SHEET

MATH 116 - FINAL EXAM - SPRING 2005

Thursday, May 5, 2005 – 8:00 AM – 12:00 NOON

PRINT NAME: _____

IN LARGE CAPITALS Last, First name

DETACH THIS ANSWER SHEET from the exam and fill in all the required information printing in LARGE CAPITAL LETTERS on both the front cover sheet of the exam and on this answer sheet.

QUESTION	ANSWER					QUESTION	ANSWER				
1.	A	B	C	D	E	13.	A	B	C	D	E
2.	A	B	C	D	E	14.	A	B	C	D	E
3.	A	B	C	D	E	15.	A	B	C	D	E
4.	A	B	C	D	E	16.	A	B	C	D	E
5.	A	B	C	D	E	17.	A	B	C	D	E
6.	A	B	C	D	E	18.	A	B	C	D	E
7.	A	B	C	D	E	19.	A	B	C	D	E
8.	A	B	C	D	E	20.	A	B	C	D	E
9.	A	B	C	D	E	21.	A	B	C	D	E
10.	A	B	C	D	E	22.	A	B	C	D	E
11.	A	B	C	D	E	23.	A	B	C	D	E
12.	A	B	C	D	E	24.	A	B	C	D	E
						25.	A	B	C	D	E

SCORE: _____

NAME(PRINT LARGE CAPITALS):

FINAL EXAM MATH 116 DUPRÉ SPRING 2005

1. Suppose that f is a continuous function on the interval $a \leq x \leq b$ and that we define the function F on the same interval by

$$F(w) = \int_a^w f(x)\,dx,$$

for each number w satisfying $a \leq w \leq b$. Then it must be true that

A) $f'(x) = F(x)$, for each x with $a \leq x \leq b$
B) $f'(c) = F(c)$, for some x with $a \leq x \leq b$
C) $F'(a) = F'(b)$
D) $F'(x) = f(x)$, for each x with $a < x < b$
E) NOT.

2. Suppose that a plane region is changing with time, and $R(t)$ is the region at time t, for all t in the interval $0 \leq t \leq 3$. Suppose it is growing because only part of its smooth boundary, say $B(t)$, at time t is moving out at velocity $v(t)$, at each point, with the velocity being perpendicular to the smooth boundary curve at each point of $B(t)$, but not depending on the particular point, that is, v is a function of t alone. Assume the rest of the boundary remains fixed. Let $A(t)$ be the area of the region $R(t)$ and let $L(t)$ be the length of $B(t)$, the part of the boundary which is moving out at time t. Assume that $v(t)$ and $L(t)$ are both continuous functions and that $A(t)$ is a differentiable function. Then:

A) $\int_0^2 L(t)\,dt = A(2)$ **B)** $\int_0^2 [L(t)]v(t)\,dt = A(2) - A(0)$
C) $A'(2) = L(0)v(2)$ **D)** $A'(0) = L(2)v(0)$ **E)** NOT.

3. Suppose that a solid region is changing with time, and $R(t)$ is the region at time t, for any t with $0 \leq t \leq 3$. Suppose it is growing because a part of its smooth boundary, say $B(t)$, at time t is moving out at velocity $v(t)$, at each point, with the velocity being perpendicular to the smooth boundary surface at each point of $B(t)$, but not depending on the particular point, that is, v is a function of t alone. Assume the rest of the boundary remains fixed. Let $V(t)$ be the volume of the region $R(t)$ and let $A(t)$ be the area of $B(t)$, at time t. Assume that $v(t)$ and $A(t)$ are continuous functions and that $V(t)$ is a differentiable function. Then:

A) $\int_0^2 A(t)\,dt = V(2)$ **B)** $\int_0^2 v(t)\,dt = V(2) - V(0)$

C) $V'(2) = A(2)v(2)$ **D)** $V'(0) = A(2)v(0)$ **E)** NOT.

4. Suppose that f and g are functions which are differentiable on the interval $[0, 5]$ and that $f(2) = 3$ and $g(3) = 2$. Suppose that $h(x) = f(g(x))$ for every x in $[0, 5]$ for which this makes sense, and in particular, for $x = 3$. Then it must be true that

A) $h'(3) = 3$ **B)** $h'(3) = f'(3)g'(3)$ **C)** $h'(2) = f'(3)g'(2)$
D) $h'(3) = f'(2)g'(3)$ **E** NOT.

For the next 10 questions, the only possible answers are:

A) $\frac{1}{1+x^2}$ **B)** $\frac{x}{\sqrt{1-x^2}}$ **C)** $\frac{1}{x\sqrt{x^2-1}}$ **D)** $\frac{1}{\sqrt{1-x^2}}$ **E)** NOT.

5. $\arcsin'(x) =$
6. $\arctan'(x) =$
7. $\arccos'(x) =$
8. $\operatorname{arccot}'(x) =$
9. $\operatorname{arcsec}'(x) =$
10. $[5 - \arccos(x)]' =$
11. $\tan(\arcsin(x)) =$
12. $\sec(\arcsin(x)) =$
13. $\csc(\operatorname{arcsec}(x) =$
14. $\arcsin(1 - \cos^2 x) =$

15. If B is a nonzero real number, then $\int_0^B x^4 e^x \, dx =$

A) $e^B(B^4 - 4B^3 + 12B^2 - 24B + 24) - 24$
B) $e^B(4B^3 - 12B^2 + 24B - 24) + 24$
C) $e^B(B^4 - 4B)$
D) $e^B(B^4 - 4B^3 + 12B^2 - 24B + 24)$
E) NOT.

16. A cylindrical tin can of height H and radius R is sitting on a laboratory bench and is filled with explosive. At the instant the explosives are detonated, the vertical sides begin moving radially out with velocity u and the flat top begins moving up with velocity v different from u. At the instant the explosives detonate, the rate of increase of volume of the cylinder is

A) $2\pi RHu + \pi R^2 v$ **B)** $\pi R^2 Hu + 2\pi Rv$ **C)** $\pi R^2 H(u + v)$
D) $2\pi Ru + Hv$ **E)** NOT.

17. The area of the region trapped between the curves $y = \sin x$ and $y = \cos x$ over the interval $[0, \pi/4]$ is equal to

A) $\sin(\pi/4)$ **B)** 1 **C)** $\cos(\pi/4) - 1$ **D)** $2\cos(\pi/4) - 1$ **E)** NOT.

18. An antiderivative of $f(x) = e^{\sec x} \sec x \tan x$ is

A) $e^{\sec x \tan x}$ **B)** $(\sec^2 x)e^{\tan x}$ **C)** $e^{\sec x}(\sec x \tan x)^2$

D) $e^{\sec x}$ **E)** NOT.

19. The region trapped between the curves $y = \cos x$ and $y = \sin x$ over the interval $[0, \pi/2]$ is rotated around the $x-$axis. The volume of the resulting solid of revolution is

A) π **B)** $\pi/2$ **C)** 2 **D)** 0 **E)** NOT.

20. The region $1 \leq y \leq xe^{x^3}$, $1 \leq x \leq 2$ is revolved around the $y-$axis. The volume of the resulting solid of revolution is

A) $\displaystyle\int_1^2 2\pi(xe^{x^3} - 1)\,dx$ **B)** $\frac{2}{3}\pi[e(e-1) - 9]$ **C)** $\frac{2}{3}\pi[e^7(e-1) - 9]$

D) $\frac{2}{3}\pi[e^7(e-1)]$ **E)** NOT.

21. The solid region R is next to the $x-$axis between planes perpendicular to that axis, the planes passing through points $x = 1$ and $x = 5$. For any point x on the axis between $x = 1$ and $x = 5$, the plane through x will cut through the region R making a cross-section of area $A(x) = x^2 \ln x$. The volume of the solid region R is

A) $\displaystyle\int_1^5 2\pi(x^2 \ln x)\,dx$ **B)** $\displaystyle\int_1^5 2\pi(x^3 \ln x)\,dx$ **C)** $\displaystyle\int_1^5 (x^2 \ln x)\,dx$

D) $\displaystyle\int_1^5 \pi(x^6 \ln^2 x)\,dx$ **E)** NOT.

22. Twenty miles of beach front property on Lake Wobegon is eroding at the rate of 3 miles per century and at the same time, the swamp on the other side of the lake is being filled in so as to cause the ten mile stretch of lake boundary along the swamp to move into the lake along the swamp boundary at the rate of 0.05 miles per year. The rate of increase of area of Lake Wobegon due to these changes is, in square miles per century

A) $(20)(3) + (10)(.05)$ **B)** $(20)(3) + (10)(5)$ **C)** $(20)(3) - (10)(.05)$

D) $(20)(3) - (10)(5)$ **E)** NOT.

23. If $f(x)$ is continuous on the whole $x-$axis, then it must be true that

A) $\displaystyle\int_0^5 2xe^{x^2} f(e^{x^2})\,dx = \int_1^{e^{25}} f(u)\,du$

B) $\displaystyle\int_0^5 2xe^{x^2} f(e^{x^2})\,dx = \int_0^{e^{25}} f(u)\,du$

C) $\displaystyle\int e^{x^2} f(e^{x^2})\,dx = \int f(u)\,du\Big|_{u=e^{x^2}}$

D) $\displaystyle\int 2e^{x} f(e^{x^2})\,dx = \int f(u)\,du\Big|_{u=e^{x^2}}$

E) NOT.

24. A nonlinear spring exerts a force of $K\tan(x/L)$ pounds when the displacement is x feet from its equilibrium position. The work done in stretching this spring from $x = 0$ out to $x = \pi L/4$ feet, in foot pounds, is

A) $\displaystyle\int_0^{\pi L/4} K\tan(u)\,du$ **B)** $\displaystyle K\int_0^{\pi L/4} \tan(u)\,du$ **C)** $\displaystyle KL\int_0^{\pi/4} \tan(u)\,du$

D) $KL(\pi/2)\ln(\sqrt{2})$ **E)** NOT.

25. A grain elevator is in the shape of a vertical cylinder of radius R, with hemispherical cap. The top of the cylinder is at height H, so the height of the whole elevator itself is $R + H$. It is filled to height H with grain of weight density ρ. The work that would be required to empty the elevator by lifting the grain over the top edge of the cylinder at height H is

A) $\pi\rho RH$ **B)** $(\pi/2)\rho RH^2$ **C)** $(\pi/2)\rho R^2H^2$

D) $(\pi/2)\rho R^2H$ **E)** NOT.