Instructor's name:________________________ **Your name:**________________________

MATH 221 FINAL EXAMINATION
Thursday, May 5, 2005, 8:00 A.M. – 12:00 NOON

INSTRUCTIONS: There is a total of 25 problems, each is worth 4 points. Do all 25 problems, and give only one best answer to each problem. It is to your advantage to answer all problems because you are not penalized for wrong answers.

The first 24 problems are multiple choice. Circle completely and carefully one best answer to each of these problems on the answer sheet. For the last problem **(Problem 25)**, fill in your answer in the blank provided. Turn in the entire exam.

1.	a	b	c	d	e	13.	a	b	c	d	e
2.	a	b	c	d	e	14.	a	b	c	d	e
3.	a	b	c	d	e	15.	a	b	c	d	e
4.	a	b	c	d	e	16.	a	b	c	d	e
5.	a	b	c	d	e	17.	a	b	c	d	e
6.	a	b	c	d	e	18.	a	b	c	d	e
7.	a	b	c	d	e	19.	a	b	c	d	e
8.	a	b	c	d	e	20.	a	b	c	d	e
9.	a	b	c	d	e	21.	a	b	c	d	e
10.	a	b	c	d	e	22.	a	b	c	d	e
11.	a	b	c	d	e	23.	a	b	c	d	e
12.	a	b	c	d	e	24.	a	b	c	d	e

25. ______________________________________

1. The projection of vector $\vec{a}$ onto $\vec{w}$ is

   (a) $\vec{a}\cdot\dfrac{\vec{w}}{|\vec{w}|}$ (b) $\vec{a}\cdot\vec{w}$ (c) $\left(\vec{a}\cdot\dfrac{\vec{w}}{|\vec{w}|}\right)\vec{w}$ (d) $\left(\vec{a}\cdot\dfrac{\vec{w}}{|\vec{w}|}\right)\dfrac{\vec{w}}{|\vec{w}|}$ (e) None of the above

2. The distance from the point $P(4,1,7)$ to the line m where $m:(x,y,z)=(1,0,5)+t(6,3,2)$ equals

   (a) 0 (b) $\sqrt{\dfrac{61}{49}}$ (c) $\dfrac{25}{7}$ (d) $\sqrt{14}$ (e) None of the above

3. Let α be the plane passing through $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$. Which one of the following vectors is normal to α?

   (a) (a,b,c) (b) (c,b,a) (c) (bc,ac,ab) (d) (ab,bc,ac)

   (e) None of the above

4. Which of the following is true?.

   (a) $\vec{b}\cdot(\vec{a}\times\vec{b})=0$ (b) $\vec{b}\cdot(\vec{a}\times\vec{b})=(\vec{b}\times\vec{a})\cdot\vec{b}$ (c) $\vec{b}\cdot(\vec{a}\times\vec{b})$ is perpendicular to both $\vec{a}$ and $\vec{b}$

   (d) $\vec{b}\cdot(\vec{a}\times\vec{b})=|\vec{b}|^2\cdot|\vec{a}|$ (e) None of the above

5. Let $f(x,y)=7x+2y+3$. Among all directional derivatives of f at $P(1,1)$, the largest is

   (a) 9 (b) $\dfrac{9}{\sqrt{2}}$ (c) $\sqrt{53}$ (d) $-\sqrt{53}$ (e) None of the above

6. Let $f(x,y,z)=xy^2z+3$, and let S be the level surface of $f(x,y,z)$ that passes through $P(1,1,1)$. Then the tangent plane of S at $P(1,1,1)$ is given by

   (a) $x+2y+z=4$ (b) $x+2y+z=7$ (c) $(x-1)+2(y-1)+(z-1)+3=0$

   (d) $-(x-1)-2(y-1)+(z-1)=0$ (e) None of the above

7. Let $f(x,y)=x^4+y^4-4xy+1$. Then

   (a) f has a local minimum at $(1,1)$ and $(-1,-1)$ and a saddle point at $(0,0)$

   (b) f has a local minimum at $(1,1)$, a local maximum at $(-1,-1)$, and a saddle point at $(0,0)$,

   (c) f has a local maximum at $(1,1)$, a local minimum at $(-1,-1)$, and a saddle point at $(0,0)$,

   (d) f has a local minimum at $(1,1)$ and $(-1,-1)$, and a local maximum at $(0,0)$,

   (e) None of the above

8. The maximum of $f(x,y) = x - 2y + 1$ subject to the constraint $(x-6)^2 + (y-1)^2 = 1$ is

   (a) $5-\sqrt{5}$ (b) $\sqrt{5}+5$ (c) 10 (d) 20 (e) None of the above

9. Let $a(t)$ be a function with $a'(0) = 2$, and let $f(x,y) = a(xy-2)$. Then the partial derivative $f_x(1,2)$ equals

   (a) 4 (b) 1 (c) 2 (d) 0 (e) None of the above

10. The linearization of $f(x,y) = x^2 + 2y^2$ at $P(-1,1)$ equals

    (a) $L(x,y) = 3 - 2(x+1) + 4(y-1)$
    (b) $L(x,y) = 1 - 2(x+1) + 2(y-1)$
    (c) $L(x,y) = -2(x+1) + 4(y-1)$
    (d) $L(x,y) = 3 - 2x + 4y$
    (e) None of the above

11. The volume of the solid which is below the graph of $z = x^2 + y^2$ and above the square $0 \le x \le 1$, $0 \le y \le 1$ on the xy-plane is

    (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{\pi}{2}$ (e) None of the above

12. Let D be the triangle region in the xy-plane bounded by the lines: $x = y$, $x = 5$, and $y = 0$. Which one of the following statements best describes the double integral $\iint\limits_D \cos(x^2y)\,dA$?

    (a) $\iint\limits_D \cos(x^2y)\,dA = \int_0^5 \int_0^x \cos(x^2y)\,dy\,dx$

    (b) $\iint\limits_D \cos(x^2y)\,dA = \int_0^5 \int_y^5 \cos(x^2y)\,dx\,dy$

    (c) $\iint\limits_D \cos(x^2y)\,dA = \int_0^5 \int_0^y \cos(x^2y)\,dx\,dy$

    (d) Both (a) and (b) are correct.

    (e) All of (a), (b), (c) are correct.

13. The double integral $\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy$ equals

(a) $\left(1+\frac{1}{e}\right)$ (b) $\frac{1}{2}\left(1-\frac{1}{e}\right)$ (c) $\frac{1}{2}\left(1+\frac{1}{e}\right)$ (d) $\frac{1}{e}$ (e) None of the above

14. The triple integral $\iiint\limits_{\Omega} \sqrt{x^2+y^2+z^2}\, dV$, where Ω is a solid region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=9$, equals

(a) 40π (b) 80π (c) 81π (d) $\frac{104}{3}\pi$ (e) None of the above

15. Let K be the tetrahedron bounded by the 3 coordinate planes and the plane $x+y+z=1$. Then the triple integral $\iiint\limits_{K} z\, dV$ equals

(a) $\frac{1}{48}$ (b) $\frac{1}{12}$ (c) $\frac{1}{24}$ (d) $\frac{1}{8}$ (e) None of the above

16. Let $\vec{F}$ be a 2-dimensional vector field defined in part of the plane. Which of the following statements is **not** always true?

(a) If $\vec{F}$ is conservative, then line integrals of $\vec{F}$ are path-independent.

(b) If $\operatorname{Curl} \vec{F} = 0$, then $\vec{F}$ is conservative.

(c) If the circulations of $\vec{F}$ around any closed curve is zero, then $\vec{F}$ is conservative.

(d) If $\vec{F}$ is conservative, then $\operatorname{Curl} \vec{F} = 0$.

(e) If line integrals of $\vec{F}$ are path-independent, then $\vec{F}$ is conservative.

17. The vector field $\vec{F}$ in the plane is given by

$$\vec{F} = \operatorname{grad}(x^2y + y^3 + 5)$$

and C is the curve $(x,y) = (t^3, t^4)$ from the origin to the point $P(1,1)$. Then $\int_C \vec{F} \cdot \vec{dr}$ equals

(a) 3 (b) 5 (c) 2 (d) 0 (e) None of the above

18. Let S be the parallelogram spanned by vectors $\vec{p} = (-2, 0, 2)$ and $\vec{q} = (-2, 2, 0)$, which is oriented by a normal vector whose z-component is positive. Let $\vec{F} = (-2, 3, 7)$ be the constant vector field. Then the flux of $\vec{F}$ across S equals

(a) 32 (b) $\dfrac{8}{\sqrt{3}}$ (c) -32 (d) -2 (e) None of the above

19. The part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 1$ has area equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{6}\left(5\sqrt{5} - 1\right)$ (c) $\frac{\pi}{4}\left(5\sqrt{5} - 1\right)$ (d) $\frac{\pi}{4}$ (e) None of the above

20. Let $\vec{F} = (x, y, z)$ and C be the straight line segment from the origin to $P(1, 2, 3)$. Then $\int_C \vec{F} \cdot d\vec{r}$ equals

(a) 7 (b) $\dfrac{\sqrt{14}}{2}$ (c) 0 (d) -7 (e) None of the above

21. Let S be the flat surface on the plane $x + y + z = 1$ which is bounded by the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. Orient S by a normal vector whose z-component is positive. Let $\vec{F} = (x, y, z)$. Then $\iint_S \vec{F} \cdot d\vec{A}$ equals

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) None of the above

22. Let C be the circle of radius 1 centered at the origin in the xy-plane which is oriented counterclockwise. Then the line integral $\int_C (\tan^{-1}\sqrt{x} + y)\, dx + (\sin^4 y + 3x)\, dy$ equals

(a) π (b) 2π (c) 3π (d) 4π (e) None of the above

23. Let $\vec{F} = (x, 2y, 3z)$ and S be the upper sphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, which is oriented by upward normals. Then $\iint_S \vec{F} \cdot d\vec{A}$ equals

(a) 2π (b) 4π (c) π (d) 6π (e) None of the above

24. Let S be the surface of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane. Let S be oriented by the outward normals, and let $\vec{F} = (x^3, 2xz^2, 3y^2 z)$. Then $\iint_S \vec{F} \cdot d\vec{A}$ equals

(a) 32π (b) 16π (c) 64π (d) 0 (e) None of the above

25. A potential of $\vec{F} = (yz+1, xz+2, xy+3)$ is given by ______________________________