ANSWER SHEET **MATH 115 FINAL EXAM**

Name, section:_____ Instructor: _____

Wednesday, December 13, 2006 – 8:00 AM – 12:00 NOON

1. Do NOT SEPARATE answer sheet from rest of test.
2. Work and CIRCLE the answer to each problem INSIDE this test.
3. Circle your answer a SECOND TIME on this page.
4. Blank answers are considered INCORRECT.
5. Do all **39** problems.
6. Turn in ENTIRE TEST.

QUESTION	ANSWER	QUESTION	ANSWER
1.	[a] [b] [c] [d] [e]	14.	[a] [b] [c] [d] [e]
2.	[a] [b] [c] [d] [e]	15.	[a] [b] [c] [d] [e]
3.	[a] [b] [c] [d] [e]	16.	[a] [b] [c] [d] [e]
4.	[a] [b] [c] [d] [e]	17.	[a] [b] [c] [d] [e]
5.	[a] [b] [c] [d] [e]	18.	[a] [b] [c] [d] [e]
6.	[a] [b] [c] [d] [e]	19.	[a] [b] [c] [d] [e]
7.	[a] [b] [c] [d] [e]	20.	[a] [b] [c] [d] [e]
8.	[a] [b] [c] [d] [e]	21.	[a] [b] [c] [d] [e]
9.	[a] [b] [c] [d] [e]	22.	[a] [b] [c] [d] [e]
10.	[a] [b] [c] [d] [e]	23.	[a] [b] [c] [d] [e]
11.	[a] [b] [c] [d] [e]	24.	[a] [b] [c] [d] [e]
12.	[a] [b] [c] [d] [e]	25.	[a] [b] [c] [d] [e]
13.	[a] [b] [c] [d] [e]	26.	[a] [b] [c] [d] [e]

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27. [a] [b] [c] [d] [e]

28. [a] [b] [c] [d] [e]

29. [a] [b] [c] [d] [e]

30. [a] [b] [c] [d] [e]

31. [a] [b] [c] [d] [e]

32. [a] [b] [c] [d] [e]

33. [a] [b] [c] [d] [e]

34. [a] [b] [c] [d] [e]

35. [a] [b] [c] [d] [e]

36. [a] [b] [c] [d] [e]

37. [a] [b] [c] [d] [e]

38. [a] [b] [c] [d] [e]

39. [a] [b] [c] [d] [e]

Math 115 FINAL EXAM — Fall 2006

Do NOT unstaple the exam. Clearly mark one answer only on the inside of the exam as well as on the cover sheet. If two or more answer are marked, no credit will be given. No partial credit will be given if a wrong answer is marked. Answer all questions, there is no penalty for wrong answers. Calculators are allowed. Give exact answers– NOT calculator approximations.

1. The solution of $|x-2|<1$ is:

(a) $-3 \leq x \leq 3$ (b) $-3 < x < 3$ (c) $(1,3)$ (d) $[1,3]$ (e) none of the above

2. The distance from the point $(1,4)$ to the point $(-2,8)$ is:

(a) $\sqrt{17}$ (b) $\sqrt{153}$ (c) 7 (d) 5 (e) none of the above

3. The equation of the line that goes through the point $(1,3)$ and is perpendicular to $y=(1/2)x+15$ is:

(a) $y=2x+5$ (b) $y=(1/-2)x+17$ (c) $y=-2x+5$ (d) $y=3x$

(e) none of the above.

4. The equation of the line that goes through the points $(1,2)$ and $(4,8)$ is:

(a) $y=2x$ (b) $y=-2x+16$ (c) $y=-2x$ (d) $y=3x-1$

(e) none of the above

5. The x-coordinate of a local maximum of $f(x)=-2x^3+3x^2+12x+5$ is:

(a) 1 (b) -1 (c) 2 (d) 6 (e) none of the above

6. For the piecewise function $f(x) = \begin{cases} \frac{2}{x}, & x < 2 \\ 3, & x = 2 \\ 3x - 5, & x > 2 \end{cases}$

(a) $\lim_{x \to 2} f(x) = 1$ and $f'(2) = -\frac{1}{2}$ (b) $\lim_{x \to 2} f(x) = 1$ and $f'(2) = 3$

(c) $\lim_{x \to 2} f(x) = 3$ and $f'(2) = -\frac{1}{2}$ (d) $\lim_{x \to 2} f(x) = 3$ and $f'(2) = 3$

(e) none of the above

7. The equation of the line that is perpendicular to the line $y = 2x + 6$ and goes through the point $(4, -10)$ is:

(a) $y = 2x - 18$ (b) $y = -2x - 2$ (c) $y = \frac{1}{2}x - 12$ (d) $y = -\frac{1}{2}x - 8$

(e) none of the above

8. The functions $f(x) = x^{3/2}$, $g(x) = -3x + 5$, and $h(x) = x^3$ are defined for all real numbers x. Which functions are one-to-one?

(a) f and g (b) g and h (c) f and h (d) only g

(e) none of the above

9. Let $g(x)$ be the inverse function $f(x) = \frac{x+3}{x-2}$. Then $g(2)$ equals:

(a) 7 (b) ∞ (c) $\frac{3}{5}$ (d) $g(2)$ does not exist

(e) none of the above

10. An interval on which the function $f(x) = -x^4 + 4x^3 + 100$ is concave up is:

(a) $(3, \infty)$ (b) $(2, \infty)$ (c) $(0, 2)$ (d) $(-\infty, 0)$ (e) none of the above

11. For the piecewise function $f(x) = \begin{cases} 3x + 2, & x < 1 \\ 6 - x, & x \geq 1 \end{cases}$

(a) $f'(1) = -1$

(b) $f'_-(1) = 3$ and $f'_+(1) = -1$

(c) $f'_-(1)$ does not exist and $f'_+(1) = -1$

(d) neither $f'_-(1)$ nor $f'_+(1)$ exist

(e) none of the above

12. The line L has slope 4 and contains the point $(-1, -6)$. Which of the following points is also on L?

(a) $(2, 8)$ (b) $(4, 18)$ (c) $(1, 4)$ (d) $(1, 2)$ (e) none of the above

13. The functions $f(x) = x^6 - 2x^2 - 3$, $g(x) = \dfrac{|x|}{1 + x^2}$, and $h(x) = x^2 + 8x$ are defined for all real numbers x. Which functions are even?

(a) f and g (b) g and h (c) f and h (d) only f

(e) none of the above

14. The function $g(x) = -x^4 + 24x^2 + 64$ has an inflection point at:

(a) $x = -1$ (b) $x = -2$ (c) $x = \sqrt{12}$ (d) $x = -\sqrt{12}$

(e) none of the above

15. The limit $\lim_{h\to 0} \dfrac{\sqrt{5+h}-\sqrt{5}}{h}$ is equal to:

(a) $5\sqrt{5}$ (b) 10 (c) $\dfrac{1}{2\sqrt{5}}$ (d) does not exist

(e) none of the above

16. Given $f(x) = \dfrac{x^2}{x^2+2x+4}$. Then $f'(1)$ is equal to:

(a) $\dfrac{12}{49}$ (b) $-\dfrac{12}{49}$ (c) $\dfrac{10}{49}$ (d) 0 (e) none of the above

17. For which x is the function $f(x) = x^3 + 3x^2 - 24x + 1$ decreasing?

(a) $x > 2$ (b) $x < 0$ (c) $-4 < x < 2$ (d) $1 < x < 3$

(e) none of the above

18. Which of the following intervals contains a solution of the equation $x^3 + x^2 - x = 2$?

(a) $(-2,-1)$ (b) $(-1,0)$ (c) $(0,1)$ (d) $(1,2)$ (e) none of the above

19. The x-coordinate of a point (determined by the Mean Value Theorem) where the function $f(x) = x + \dfrac{9}{x}$ on the interval $[1,3]$ (i.e. for $0 < x < 3$) has a tangent line which is parallel to the secant line through (1,0) and (3,6) is :

(a) 2 (b) -2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$ (e) none of the above

20. The x-coordinate of a point (determined by the Mean Value Theorem) where the function $f(x) = \sqrt{x+1}$ on the interval $[0, 8]$ (i.e. for $0 < x < 8$) has a tangent line which is parallel to the secant line through (0,1) and (8,3) is :

(a) 3 (b) -2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$ (e) none of the above

21. Consider the following three limits: **(i)** $\lim_{x \to -\infty} x^{-3}$, **(ii)** $\lim_{x \to -\infty} x^{\frac{1}{3}}$, and **(iii)** $\lim_{x \to 1+} \dfrac{x^2 - 3x + 2}{x^2 - 1}$. Which of them are either infinite, or negative infinite?

(a) **(i)** and **(ii)** (b) **(ii)** and **(iii)** (c) (i) and **(iii)** (d) only **(ii)**

(e) none of the above

22. Given $f(x) = \dfrac{x^2}{x+1}$. Which of the following is true?

(a) $x = 0$ is a local minimum point and $x = -2$ is a local maximum point

(b) $x = 0$ is a local maximum point (c) $x = 2$ is a local maximum point

(d) $x = 4$ is a local minimum point (e) none of the above

23. The limit $\lim_{x \to 1} \dfrac{x^2 - 2x}{(x-1)^2}$ is equal to:

(a) -2 (b) $-\infty$ (c) ∞ (d) 1 (e) none of the above

24. The limit $\lim\limits_{x\to 1} \dfrac{\frac{2}{x+1} - 1}{x - 1}$ is equal to:

(a) $\dfrac{1}{4}$ (b) $-\dfrac{1}{2}$ (c) -1 (d) 1 (e) none of the above

25. Given $f(x) = \sqrt{x^4 - 4x + 1}$. Then $f'(0)$ is equal to:

(a) -1 (b) -2 (c) 1 (d) 2 (e) none of the above

26. The equation of the tangent line to $x^3 + y^3 + y = 1$ at the point $(-1, 1)$ is:

(a) $y = \frac{1}{4}x + \frac{5}{4}$ (b) $y = -\frac{3}{4}x + \frac{1}{4}$ (c) $y = -x$ (d) $y = 3x + 4$

(e) none of the above

27. The limit $\lim_{x \to 1} \frac{x^2 - x}{x^2 + x - 2}$ is equal to:

(a) $\frac{1}{3}$ (b) ∞ (c) 1 (d) does not exist (e) none of the above

28. Given $f(x) = (x^2 + 4)^{10}$. Then its second derivative $\frac{d^2 f}{dx^2}$ is equal to:

(a) $90(x^2+4)^8$ (b) $(380x^2+80)(x^2+4)^8$ (c) $360x^2(x^2+4)^8$ (d) $20x(x^2+4)^9$

(e) none of the above

29. The global maximum of the function $f(x) = -x^2 + 4x$ on the interval $3 \leq x \leq 4$ is at:

(a) $x = 3.5$ (b) $x = 2$ (c) $x = 3.75$ (d) $x = 4$ (e) none of the above

30. A complete set of vertical and horizontal asymptotes of the function $f(x) = \frac{2}{x+1} - \frac{1}{x+2}$ is:

(a) $x = -1,\ x = -2,\ y = 0$ (b) $x = -2,\ y = 1$

(c) $x = -1,\ x = -2$ (d) $x = -1,\ y = 1$ (e) none of the above

31. What are two positive numbers x and y whose product is 10 and whose sum is as small as possible?

(a) $x = 2,\ y = 5$ (b) $x = 1,\ y = 10$ (c) $x = \sqrt{10},\ y = \sqrt{10}$

(d) the problem has no solution (e) none of the above

32. Air is pumped into a ball at a rate of 10 cubic inches per second. How fast is the radius of the ball increasing at the instant that the ball has a radius of 2 inches? $(V = (4\pi r^3)/3)$

(a) 2 in/sec (b) $\frac{2}{\pi}$ in/sec (c) $\frac{5}{8\pi}$ in/sec (d) $\frac{5}{2\pi}$ in/sec

(e) none of the above

33. The area of the largest rectangle that can be inscribed under the graph of the function $y = f(x) = 3 - x^2$ with the base along the x-axis is

(a) 4 (b) 1 (c) $4\sqrt{(3)}$ (d) $4\sqrt{(3)}$ (e) none of the above

The following refers to the next two problems. A square sheet of cardboard 12 inches on a side is to be used to make an open-top box by cutting a small square of cardboard from each corner and bending up the sides. What is the length x of the side of the small square for the box to have as large a volume $V = V(x)$ as possible ?

34. V(x) equals :

(a) $144x$ (b) $x(12 - 2x)^2$ (c) $x(12 - x)$ (d) $x(12 - x)^2$ (e) none of the above

35 For a maximum, in the last problem, x should be :

(a) $x = 6$ (b) $x = 4$ (c) $x = 3$ (d) $x = 2$ (e) none of the above

36. 4π cubic meters per second of water are poured into a long cylindrical container of radius 1 meter. How fast in meters per second does the water level in the cylinder rise ?

(a) 4 (b) 1.5 (c) 2 (d) 2.5 (e) none of the above

37. The foot of a 5 foot ladder, which is leaning against a wall is being dragged away from the wall at the rate of 8 feet per second. At that instant of time when the foot of the ladder is 4 feet from the wall, how fast (in feet per second) is the top of the ladder is descending toward the floor ?

(a) 4 (b) $\frac{32}{3}$ (c) 2 (d) $\frac{8}{3}$ (e) none of the above

The following refers to the next two problems. What should the radius r be of a 27 cubic inch cylindrical can with a top be equal to, if the surface area $A = A(r)$ is a minimum ?

38. A(r) equals :

(a) $2\pi r^2+(27/r)$ (b) $2\pi r^2+(54/r)$ (c) $\pi r^2+(32/\pi r)$ (d) $2\pi r^2+(54/\pi r)$
(e) none of the above

39. The minimum of $A(r)$, $0 < r < \infty$ occurs when r equals: :

(a) $\frac{3}{(2\pi)^{1/3}}$ (b) $\frac{9}{(\pi)^{1/3}}$ (c) $\frac{27}{(2\pi)^{2/3}}$ (d) $\frac{9}{(2\pi)^{1/3}}$

(e) none of the above