

your SSN: _____

your name: _____

CALCULUS I (MATH 121)

Final exam

FALL 2006

Instructions: There is no partial credit. Only the answers written in the provided box will be graded. Empty boxes will receive zero points. Your exam should have 7 pages.

For grading only. Do not write in this space.

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|-----------|-----------|
| 1. _____ | 13. _____ |
| 2. _____ | 14. _____ |
| 3. _____ | 15. _____ |
| 4. _____ | 16. _____ |
| 5. _____ | 17. _____ |
| 6. _____ | 18. _____ |
| 7. _____ | 19. _____ |
| 8. _____ | 20. _____ |
| 9. _____ | 21. _____ |
| 10. _____ | 22. _____ |
| 11. _____ | 23. _____ |
| 12. _____ | 24. _____ |
| | 25. _____ |

Total: _____ /180

1. [5pt] Find the derivative of $f(x) = x^3 e^{2x}$

$$f'(x) =$$

2. [5pt] Find the derivative of $f(x) = \ln(3x + \cos(x))$

$$f'(x) =$$

3. [5pt] Find the derivative of $f(x) = 5 \left(\frac{x}{x^2 + 1} \right)^4$

$$f'(x) =$$

4. [5pt] Find the derivative of $f(x) = (3x^3 + x - 1)^2 (x^2 + 2)^3$

$$f'(x) =$$

5. [5pt] Find the derivative of $f(x) = e^{x^2}$

$$f'(x) =$$

6. [5pt] Find the derivative of $f(x) = 4x^2 \ln(1/x)$

$$f'(x) =$$

7. [5pt] If $x \sin(y) + x^2 y^2 = ye^x$, find dy/dx .

$$\frac{dy}{dx} =$$

8. [5pt] Find the domain of $f(x) = \sqrt{x+7}$

9. [5pt] Find the limit. If it is not a finite number, indicate if it is ∞ , $-\infty$ or if it does not exist.

$$\lim_{x \rightarrow 0} \frac{1}{\cos(x)}$$

10. [5pt] Find the limit. If it is not a finite number, indicate if it is ∞ , $-\infty$ or if it does not exist.

$$\lim_{x \rightarrow 1^-} \frac{1}{\sqrt{1-x^3}}$$

11. [5pt] Find the limit. If it is not a finite number, indicate if it is ∞ , $-\infty$ or if it does not exist.

$$\lim_{x \rightarrow 0} \frac{[\sin(2x)]^2}{x^2}$$

12. [10pt] For what value of c is the following function continuous everywhere?

$$f(x) = \begin{cases} x + c & \text{for } x \leq 2 \\ \frac{x^2 - 3x + 2}{x^2 - 2x} & \text{for } x > 2 \end{cases}$$

$c =$

13. [10pt] Let A be any positive number. Find the horizontal and the vertical asymptote of

$$y = \frac{7x^5 + 3x^4 + Ax + 1}{(x^3 - 5)(x^2 + 1)}$$

Horizontal Asymptote:

Vertical Asymptote:

14. [10pt] Let A be a positive number and consider the function

$$f(x) = x^{3/5}(8A - x)$$

- (a) Find all critical numbers

Critical numbers:

- (b) For each critical number, determine if it corresponds to a local minimum, a local maximum or neither.

15. [10pt] Consider the function $f(x) = -x^3 + 8x^2 + 10x + 1$.

(a) Find all intervals where $f(x)$ is decreasing.

(b) Find all inflection points of $f(x)$.

Inflection points:

16. [10pt] Consider the function $f(x) = x^4 - 3\pi x^3$. Find all intervals where $f(x)$ is concave up.

17. [10pt] The volume of a cube is increasing at a rate of $9 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm?

Answer:

18. [10pt] A poster is to have an area of 200 in^2 with 2-inch margins at the bottom and sides and a 1-inch margin at the top. What dimensions will give the largest printed area?

Answer:

19. [10pt] Starting with $x_1 = 1$, find the second (x_2) and third (x_3) approximations to the root of the equation $x^3 - 2x - 1 = 0$ using Newton's method.

$x_2 =$

$x_3 =$

20. [5pt] Find the indefinite integral

$$\int \left(6e^{3x} + \frac{2x^3 - \sqrt{x}}{x} \right) dx$$

21. [5pt] Find the area under the graph of $f(x) = 1 + 3x + x^2$ between $x = 1$ and $x = 3$.

22. [10pt] Evaluate

$$\int_0^1 x^2 \sqrt{2 + x^3} dx$$

Answer:

23. [10pt] Evaluate

$$\int_0^1 \frac{8x}{(1-4x^2)^2} dx$$

Answer:

24. [5pt] Given the function

$$g(x) = \int_5^x \sin(t^2)e^t dt$$

find $g'(x)$.

$g'(x) =$

25. [10pt] A particle moves along a line so that its velocity at time t is $v(t) = 2t^2 - 4t - 5$ meters per second. Find the displacement of the particle during the time period $1 \leq t \leq 3$.

Answer: