Fall 2006

1. **[5pt] Find the derivative of $f(x) = x^3\, e^{2x}$**

$f'(x) =$ $3x^2 e^{2x} + 2x^3 e^{2x}$

2. **[5pt] Find the derivative of $f(x) = \ln(3x + \cos(x))$**

$f'(x) =$ $\dfrac{3 - \sin x}{3x + \cos x}$

3. **[5pt] Find the derivative of $f(x) = 5\left(\dfrac{x}{x^2+1}\right)^4$**

$f'(x) =$ $\dfrac{20\, x^3(1 - x^2)}{(x^2+1)^5}$

4. **[5pt] Find the derivative of $f(x) = (3x^3 + x - 1)^2\, (x^2 + 2)^3$**

$f'(x) =$ $2(3x^3 + x - 1)(9x^2+1)(x^2+2)^3 + 6x\,(3x^3+x-1)^2 \cdot (x^2+2)^2$

5. **[5pt] Find the derivative of $f(x) = e^{x^2}$**

$f'(x) =$ $2x\, e^{x^2}$

6. **[5pt] Find the derivative of $f(x) = 4x^2\, \ln(1/x)$**

$f'(x) =$ $8x \ln x - 4x$

7. **[5pt]** If $x\sin(y) + x^2y^2 = ye^x$, find dy/dx.

$$\frac{dy}{dx} = \frac{ye^x - \sin y - 2xy^2}{x\cos y + 2x^2y - e^x}$$

8. **[5pt]** Find the domain of $f(x) = \sqrt{x+7}$

$[-7, \infty)$

9. **[5pt]** Find the limit. If it is not a finite number, indicate if it is ∞, $-\infty$ or if it does not exist.

$$\lim_{x\to 0} \frac{1}{\cos(x)}$$

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10. **[5pt]** Find the limit. If it is not a finite number, indicate if it is ∞, $-\infty$ or if it does not exist.

$$\lim_{x\to 1^-} \frac{1}{\sqrt{1-x^3}}$$

$+\infty$

11. **[5pt]** Find the limit. If it is not a finite number, indicate if it is ∞, $-\infty$ or if it does not exist.

$$\lim_{x\to 0} \frac{[\sin(2x)]^2}{x^2}$$

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12. **[10pt] For what value of c is the following function continuous everywhere?**

$$f(x) = \begin{cases} x + c & \text{for } x \le 2 \\ \dfrac{x^2 - 3x + 2}{x^2 - 2x} & \text{for } x > 2 \end{cases}$$

$c =$ $-3/2$

13. **[10pt] Let A be any positive number. Find the horizontal and the vertical asymptote of**

$$y = \frac{7x^5 + 3x^4 + Ax + 1}{(x^3 - 5)(x^2 + 1)}$$

Horizontal Asymptote:	$y = 7$
Vertical Asymptote:	$x = \sqrt[3]{5}$

14. **[10pt] Let A be a positive number and consider the function**

$$f(x) = x^{3/5}(8A - x)$$

(a) Find all critical numbers

Critical numbers: $x = 0, 3A$

(b) For each critical number, determine if it corresponds to a local minimum, a local maximum or neither.

0: neither ; 3A: local max

15. [10pt] Consider the function $f(x) = -x^3 + 8x^2 + 10x + 1$.

(a) Find all intervals where $f(x)$ is decreasing.

$\boxed{\left(-\infty, \frac{8}{3} - \frac{\sqrt{94}}{3}\right) \text{ and } \left(\frac{8}{3} + \frac{\sqrt{94}}{3}, \infty\right)}$

(b) Find all inflection points of $f(x)$.

Inflection points: $\left(\frac{8}{3}, f\left(\frac{8}{3}\right)\right) = \left(\frac{8}{3}, \frac{1771}{27}\right)$

16. [10pt] Consider the function $f(x) = x^4 - 3\pi x^3$. Find all intervals where $f(x)$ is concave up.

$\boxed{(-\infty, 0) \text{ and } (3\pi/2, \infty)}$

17. [10pt] The volume of a cube is increasing at a rate of 9 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

Answer: 1.2 cm^2/min

18. [10pt] A poster is to have an area of 200 in^2 with 2-inch margins at the bottom and sides and a 1-inch margin at the top. What dimensions will give the largest printed area?

Answer: $\frac{20}{\sqrt{3}} \times 10\sqrt{3}$

19. [10pt] Starting with $x_1 = 1$, find the second (x_2) and third (x_3) approximations to the root of the equation $x^3 - 2x - 1 = 0$ using Newton's method.

$x_2 =$ 3
$x_3 =$ 2.2

20. [5pt] Find the indefinite integral

$$\int \left(6e^{3x} + \frac{2x^3 - \sqrt{x}}{x} \right) dx$$

$2e^{3x} - x^2 - 2x^{1/2} + C$

21. [5pt] Find the area under the graph of $f(x) = 1 + 3x + x^2$ between $x = 1$ and $x = 3$.

$\frac{68}{3}$

22. [10pt] Evaluate

$$\int_0^1 x^2\sqrt{2 + x^3}\, dx$$

Answer: $\frac{2}{9}\left(3^{3/2} - 2^{3/2}\right)$

23. **[10pt] Evaluate**

$$\int_0^1 \frac{8x}{(1-4x^2)^2}\,dx$$

Answer: $-4/3$

24. **[5pt] Given the function**

$$g(x) = \int_5^x \sin(t^2)e^t\,dt$$

find $g'(x)$.

$g'(x) =$ $\sin x^2\, e^x$

25. **[10pt] A particle moves along a line so that its velocity at time t is $v(t) = 2t^2 - 4t - 5$ meters per second. Find the displacement of the particle during the time period $1 \le t \le 3$.**

Answer: $s(4) - s(1) = \int_1^4 v(t)\,dt = \frac{-26}{3}$

i.e. $\frac{26}{3}$ m to the left