

MATH 122-01  
FALL 2006 FINAL  
EACH QUESTION WORTH 5 PTS  
SHOW ALL WORK ON SEPARATE PAPER

Name: \_\_\_\_\_

1. Find the area between the curve  $y = \sin^3(x)$  and the  $x$ -axis from  $x = 0$  to  $x = \pi$ .

a  $\frac{4}{3}$                        b 0                       c  $-2$   
 d  $\frac{1}{3}$                        e None of these

2. Let  $b$  be a nonzero real number. Evaluate  $\int_0^1 x^2 \sin(bx) dx$ .

a  $b \sin(b)$   
 b  $\frac{2 \sin b}{b^2} + \frac{2 \cos b}{b^3} - \frac{\cos b}{b} - \frac{2}{b^3}$   
 c  $\frac{2 \sin b}{b^2} - \frac{2 \cos b}{b^3} - \frac{\cos b}{b} - \frac{2}{b^3}$   
 d  $\frac{2 \sin b}{b^2} - \frac{2 \cos b}{b^3} + \frac{\cos b}{b} + \frac{2}{b^3}$   
 e None of these

3. Let  $R$  be the region enclosed by  $y = x(1-x)$  and the  $x$ -axis. Calculate the volume of the solid obtained by rotating  $R$  about the  $x$ -axis.

a  $\frac{\pi}{30}$                        b  $\frac{2\pi}{15}$                        c  $\frac{\pi}{6}$   
 d  $6\pi$                        e None of these

4. Given a filled cube-shaped pool of side length  $S$  (in feet), calculate the work (in ft-lb) required to pump all the water to the top of the pool. (Note: Assume the water has density  $62.5 \text{ lb/ft}^3$ ).

a  $62.5S^4$                        b  $62.5S^3$                        c  $\frac{62.5S^3}{3}$   
 d  $\frac{62.5S^4}{2}$                        e None of these

5. Let  $a > 1$ . Evaluate  $\int_1^a \frac{\ln(x)}{x} dx$ .

a  $1/a$

b  $\ln^2(a)$

c  $\ln^2(a) - 1$

d  $\frac{\ln^2(a)}{a^2}$

e None of these

6. Evaluate  $\int \frac{dx}{1+4x^2}$ .

a  $\tan^{-1}(4x) + C$

b  $\tan^{-1}(2x) + C$

c  $-\frac{1}{1+4x} + C$

d  $\tan^{-1}\left(\frac{x}{2}\right) + C$

e None of these

7. Evaluate  $\int \frac{x^2+1}{x^2+3x+2} dx$ .

a  $x - \ln|x+2| - \ln|x+1| + C$

b  $x - 5 \ln|x+2| - 2 \ln|x+1| + C$

c  $x + 5 \ln|x+2| - 2 \ln|x+1| + C$

d  $x - 5 \ln|x+2| + 2 \ln|x+1| + C$

e None of these

8. Using the Midpoint Rule with four subintervals, approximate  $\int_0^8 e^{x^2} dx$ .

a  $(e + e^9 + e^{25} + e^{49})$

b  $0.25(e + e^9 + e^{25} + e^{49})$

c  $0.25(e + e^3 + e^5 + e^7)$

d  $2(e + e^9 + e^{25} + e^{49})$

e None of these

9. Evaluate  $\int_0^\infty \frac{1}{(x+1)^2} dx$ .

a 1

b  $\tan^{-1}(\pi)$

c  $e - 1$

d 2

e None of these

10. Let  $c > 0$ . Find the arc length of the curve  $y = x^{3/2}$  from  $x = 0$  to  $x = c$ .

- a  $\frac{8}{27}((1 + \frac{9}{4}c)^{3/2} - 1)$      b  $((1 + \frac{9}{4}c)^{3/2} - 1)$      c  $\frac{8}{27}(1 + \frac{9}{4}c)^{3/2}$   
 d  $c^{3/2}$      e None of these

11. Find a solution to the differential equation  $2\frac{dy}{dx} = y \sin(x)$ .

- a  $y = e^{\frac{1}{2} \cos(x)} + C$      b  $y = e^{\frac{1}{2} \cos(x)} + C$      c  $y = Ae^{-\frac{1}{2} \cos(x)}$   
 d  $y = Ae^{\frac{1}{2} \cos(x)}$      e None of these

12. Suppose a certain radioactive isotope has a half-life of  $H$  (measured in years). How many years will it take until 90 percent of the isotope has decayed?

- a  $\frac{\ln(.1)}{\ln(.5)}$      b  $\frac{9}{5}H$      c  $\frac{\ln(.1)}{\ln(.5)}H$   
 d  $\frac{\ln(.9)}{\ln(.5)}H$      e None of these

13. Find the solution of the differential equation  $\frac{dy}{dx} + \frac{y}{x} = 1$ .

- a  $y = e^{1+\frac{1}{x}} + C$      b  $y = C - \frac{1}{x}$      c  $y = \frac{1}{2}x + C$   
 d  $y = \frac{1}{2}x + \frac{C}{x}$      e None of these

14. Convert the point  $(1, 1)$  from Cartesian coordinates to Polar coordinates.

- a  $(\sqrt{2}, \frac{\pi}{4})$      b  $(-\sqrt{2}, \frac{3\pi}{4})$      c  $(\sqrt{2}, \frac{-\pi}{4})$   
 d All of these     e None of these

15. Find the area enclosed by the polar curve given by  $r = 1 + \cos(\theta)$ .

- a  $2\pi$      b  $\pi$      c  $2 + \pi$   
 d  $1$      e None of these

16. (For the next four problems, answer "C" if the series converges and "D" if the series diverges. Each part is worth 5 points.)

(a)  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$   C converge  D diverge

(b)  $\sum_{n=1}^{\infty} \frac{n!}{2^n}$   C converge  D diverge

(c)  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$   C converge  D diverge

(d)  $\sum_{n=1}^{\infty} \frac{n^2 2^n}{n^3 3^n}$   C converge  D diverge

17. The interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{\ln(n)}{n} (x-5)^n$  is

- a Half-open on the left  b Half-open on the right  c Open on both sides  
 d Closed on both sides  e None of these

18. Which of the following power series is a representation of  $f(x) = \sin(x)$ ?

a  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

b  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$

c  $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi)^{2n+1}}{(2n+1)!}$

d All of these

e None of these