MATH 131 Final Exam Fall 2006

Wednesday, December 13, 2006, 8:00 AM – 12:00 Noon

Name:________________________________ Social Security #________________

Instructor:____

Instructions:

- Write your name, social security number, and your instructor's name clearly on this page in the spaces provided.
- DO NOT SEPARATE this page from the other pages of the test.
- Work each problem in the space provided below the problem.
- Unreadable work will not be graded.
- Incorrect answers worth 0 points, answers left blank 0 points.
- When you are finished with the exam, turn in the ENTIRE TEST.

1.	(20 points)	________	7.	(10 points)	________
2.	(10 points)	________	8.	(25 points)	________
3.	(25 points)	________	9.	(5 points)	________
4.	(25 points)	________	10.	(15 points)	________
5.	(10 points)	________	11.	(10 points)	________
6.	(15 points)	________	TOTAL (170 points)		________

1. (20 points) Find the derivative of each of the functions below:

(a) $f(x) = x\cos x$

(b) $f(x) = e^{-1/x^2}$

(c) $f(x) = \ln \frac{x}{\sqrt{x^2+1}}$

(d) $F(x) = \int_0^x \sqrt{t + \sqrt{t}}\, dt$

2. (10 points) Consider $f(x) = \sqrt[5]{x^2+1}$.

(a) Give the linear approximation $L(x)$ to $f(x)$ at $x = 0$.

(b) Use the linear approximation from part (a) to approximate $\sqrt[5]{1.01}$.

3. (25 points) Let $f(x) = \dfrac{x^3}{x^2+1}$. Its derivatives are

$$f'(x) = \frac{x^2(x^2+3)}{(x^2+1)^2}, f''(x) = \frac{-2x(x^2-3)}{(x^2+1)^3}.$$

(a) Give any vertical asymptotes and horizontal asymptotes of f. Identify the relevant limits relating to these asymptotes.

(b) Identify the intervals on which f is increasing and the intervals on which f is decreasing.

(c) Find any local maxima and local minima of f.

(d) Identify the intervals on which f is concave upward and the intervals on which f is concave downward.

(e) Find any points of inflection.

4. (25 points) Evaluate the following definite and indefinite integrals.

(a) $\int \frac{\sin x}{\sqrt{\cos x + 1}} dx$

(b) $\int_0^4 |x-2| dx$ (Hint: Interpret this in terms of the area.)

(c) $\int \frac{x}{e^{-x}}\, dx$

(d) $\int \cos^3 x \, dx$

(e) $\int \frac{dx}{x^2 - 5x + 6}$

5. (10 points) Express the following as definite integrals. DO NOT EVALUATE THESE INTEGRALS.

(a) the area enclosed by the graphs of $y = x^2$ and $y = 18 - x^2$

(b) Let R be the region in the first quadrant enclosed by the graph of $y = 2 - x$ and the coordinate axes. Express the volume of the solid obtained by revolving about the x-axis.

6. (15 points) This problem will concern the differential equation

$$y' = 3y(y-2).$$

a) If $y(x)$ is a solution, and $y(0) = 0$, then find the limit

$$\lim_{x\to\infty} y(x)$$

b) Use Euler's method with step size .25 to find an approximation to $y(1)$ for the initial condition $y(0) = 1$.

c) Find a formula for the solution to the equation, using separation of variables.

7. (10 points) Find the limit of each of the following sequences and justify your answer.

a)

$$a_n = \frac{n^2+4}{2n^2-9}$$

b)

$$b_n = \frac{\sin^2 n}{n+1}$$

8. (25 points) for each of the following infinite series, tell whether it converges or diverges. Justify your answers.

a)

$$\sum_{n=1}^{\infty} \frac{n^2+4}{2n^2-9}$$

b)

$$\sum_{n=1}^{\infty} e^{-n}$$

c)

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

d)

$$\sum_{n=1}^{\infty} \frac{n}{n^2+3n+8}$$

e) Does the series converges absolutely or conditionally?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{n!}$$

9. (5 points) Given that

$$A = \sum_{n=10}^{\infty} \frac{1}{n^{4.6}}, B = \int_{10}^{\infty} \frac{dx}{x^{4.6}}.$$

Choose the best answer and justify with reasons.
a) $A \geq B$, b) $A \leq B$, c) $A > B$, d) $A < B$, e) $A = B$.

10. (15 points) Find the radius of convergence of each of the following power series:

a)

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

b)

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

c)

$$\sum_{n=1}^{\infty} n^n x^n$$

11. (10 points) (a)Write down the Taylor series, about $x = 0$, for the function

$$f(x) = \frac{\cos x - 1}{x^2}.$$

and use your answer to evaluate the integral as an infinite series.

$$\int \frac{\cos x - 1}{x^2}$$

(b) Write out the first three terms of the Taylor series, about $x = 0$, for the functions $\cos x$ and

$$\frac{\sin x}{x},$$

and use these results to evaluate

$$\lim_{x \to 0} \frac{\frac{\sin x}{x} - \cos x}{x^2}.$$