

Instructor's Name: \_\_\_\_\_ Your Name: \_\_\_\_\_

**221 FINAL EXAMINATION**  
**Wednesday, December 13, 2006, 8:00 A.M.-12:00 NOON**

**INSTRUCTIONS:** Do ALL 24 problems. For each question, choose only one best answer. All problems carry equal weight. It may be to your advantage to guess, because you are not penalized for wrong answers. **No calculators are allowed.**

Inside the exam **CIRCLE COMPLETELY** and **CLEARLY ONE BEST ANSWER** to each problem. Then **ALSO CAREFULLY** circle your answers on the front page of the exam.

**SHOW** and **TURN IN ALL WORK** inside this exam leading to your answers. **NO WORK, NO CREDIT.** Print your **NAME** on each page of this exam. Do not separate any of the pages of the exam.

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|-----|----|----|----|----|----|-----|----|----|----|----|----|
| 1.  | a. | b. | c. | d. | e. | 13. | a. | b. | c. | d. | e. |
| 2.  | a. | b. | c. | d. | e. | 14. | a. | b. | c. | d. | e. |
| 3.  | a. | b. | c. | d. | e. | 15. | a. | b. | c. | d. | e. |
| 4.  | a. | b. | c. | d. | e. | 16. | a. | b. | c. | d. | e. |
| 5.  | a. | b. | c. | d. | e. | 17. | a. | b. | c. | d. | e. |
| 6.  | a. | b. | c. | d. | e. | 18. | a. | b. | c. | d. | e. |
| 7.  | a. | b. | c. | d. | e. | 19. | a. | b. | c. | d. | e. |
| 8.  | a. | b. | c. | d. | e. | 20. | a. | b. | c. | d. | e. |
| 9.  | a. | b. | c. | d. | e. | 21. | a. | b. | c. | d. | e. |
| 10. | a. | b. | c. | d. | e. | 22. | a. | b. | c. | d. | e. |
| 11. | a. | b. | c. | d. | e. | 23. | a. | b. | c. | d. | e. |
| 12. | a. | b. | c. | d. | e. | 24. | a. | b. | c. | d. | e. |

1. What is the area of the triangle with vertices  $P(1, 0, 0)$ ,  $Q(0, 1, 0)$ , and  $R(0, 0, 1)$ ?

- (a)  $\frac{1}{2}$       (b)  $\frac{\sqrt{2}}{2}$       (c)  $\frac{\sqrt{3}}{2}$       (d) 1      (e) None of the above

2. Suppose that  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  are three non-zero vectors satisfying  $\vec{a} \cdot \vec{b} = 2$  and  $\vec{a} \cdot \vec{c} = 0$ . Which of the following statements must be true?

- (a)  $\vec{a}$  and  $\vec{b}$  are perpendicular    (b)  $\vec{b}$  and  $\vec{c}$  are perpendicular    (c)  $\vec{a} \times \vec{b} \neq 0$   
(d)  $\vec{a} \times \vec{c} \neq 0$                       (e) None of the above

3. The angle between the two vectors  $\vec{v} = \langle 0, 1, 0 \rangle$  and  $\vec{w} = \langle \frac{\sqrt{2}}{2}, \sqrt{3}, -\frac{\sqrt{2}}{2} \rangle$  is
- (a)  $\frac{\pi}{6}$       (b)  $\frac{\pi}{4}$       (c)  $\frac{\pi}{3}$       (d)  $\frac{\pi}{2}$       (e) None of the above

4. Consider the following three statements:

- (I) Two lines perpendicular to a third line are parallel.  
(II) Two planes perpendicular to a third plane are parallel.  
(III) Two lines perpendicular to a plane are parallel.

Which of these statements is true?

- (a) (I) only      (b) (I) and (II) only      (c) (I) and (III) only  
(d) (II) and (III) only      (e) None of the above

5. A ball rolls off a horizontal table 4 feet high at a speed of 2 feet per second. What is the velocity vector of the ball when it hits the ground?

- (a)  $(0, -32)$    (b)  $(2, -8)$    (c)  $(2, -16)$    (d)  $(0, -16)$    (e) None of the above

6.  $f(x, y) = x^2y + xy^4$

At the point  $(1, 1)$ , in what direction is  $f$  increasing at the greatest rate?

- (a)  $(\frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}})$    (b)  $(\frac{2}{\sqrt{20}}, \frac{4}{\sqrt{20}})$    (c)  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$    (d)  $(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}})$    (e) None of the above

7.  $f(x, y) = x^2 e^{-3y} + 4x + y^2$

Find the directional derivative of  $f$  at the point  $(1, 0)$  in the direction  $\vec{w} = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ .

- (a) 0      (b)  $\frac{3}{\sqrt{2}}$       (c)  $-\frac{6}{\sqrt{2}}$       (d)  $-\frac{1}{\sqrt{2}}$       (e) None of the above

8. Find the equation of the plane tangent to the surface with equation  $x^2y + y^3 + z^3 + zx^3 = 0$  at the point  $\vec{p} = (1, 1, -1)$ .

- (a)  $x + y - z = 3$       (b)  $-x + 4y + 4z = -1$       (c)  $-x + 6y + 2z = 3$   
(d)  $2x - y + z = 0$       (e) None of the above

9. Let  $f(x, y, z) = z\sqrt{x+y}$ . Using the linear approximation of  $f$  at the point  $(3, 1, 1)$ , we can determine that  $f(3.1, 1.1, 0.8)$  is approximately equal to

- (a)  $-0.35$       (b)  $0.1$       (c)  $1.65$       (d)  $2.1$       (e) None of the above

10. The voltage  $V$  in a simple electrical circuit is slowly decreasing as the battery wears out, while the resistance  $R$  is slowly increasing as the resistor heats up. The current  $I$  in the circuit is related to the voltage and resistance by Ohm's Law, which states that  $V = IR$ . Use this to determine how the current is changing at the moment when  $R = 10\Omega$ ,  $I = 0.08A$ ,  $\frac{dV}{dt} = -0.06V/s$  and  $\frac{dR}{dt} = 3\Omega/s$ . [ $V$ =volts,  $\Omega$ =ohms,  $A$ =amps]

- (a) The current is decreasing at a rate of  $0.8A/s$   
(b) The current is decreasing at a rate of  $0.03A/s$   
(c) The current is increasing at a rate of  $0.018A/s$   
(d) The current is increasing at a rate of  $2.94A/s$   
(e) None of the above

11. Let  $f(x, y) = x^3 + y^2 - 6xy + 1$ . Applying the Second Derivative Test to the critical point  $(6, 18)$  tells us that

- (a)  $(6, 18)$  is a local minimum point
- (b)  $(6, 18)$  is a local maximum point
- (c)  $(6, 18)$  is a saddle point
- (d) The second derivative test is inconclusive
- (e) The point  $(6, 18)$  is not a critical point of  $f$

12. The maximum value that the function  $f(x, y, z) = x + y + z$  takes on the unit sphere  $x^2 + y^2 + z^2 = 1$  is

- (a)  $\frac{1}{\sqrt{3}}$
- (b) 1
- (c)  $\sqrt{3}$
- (d) 3
- (e) None of the above

13. Let  $S$  be the solid consisting of the top half of the ball of radius 2 with center at  $(0, 0, 0)$ . The weight density of the ball is  $d(x, y, z) = z^2 \text{ lbs/ft}^3$ . Find the total weight of  $S$ .

- (a)  $\frac{32\pi}{5}$       (b)  $32\pi$       (c)  $\frac{64\pi}{15}$       (d)  $\frac{16\pi}{5}$       (e) None of the above

14. Find the value of the double integral  $\iint_R x^3 + xy^2 dA$  where  $R$  is the quarter disc with center  $(0, 0)$ , radius 1, in the first quadrant  $[x \geq 0, y \geq 0]$ .

- (a) 1      (b)  $\frac{1}{5}$       (c)  $\frac{\pi}{4}$       (d)  $\frac{1}{4}$       (e) None of the above

15. Find the value of the iterated integral  $\int_0^{\sqrt{\pi}} \int_x^{\sqrt{\pi}} \sin(y^2) dy dx$

- (a)  $2\sqrt{\pi}$       (b)  $2\pi$       (c) 2      (d) 1      (e) None of the above

16. Find the volume of the solid in the first octant bounded by the coordinate planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and the plane  $4x + 2y + z = 8$ .

- (a)  $\frac{7}{3}$       (b)  $\frac{32}{3}$       (c)  $\frac{11}{6}$       (d) 12      (e) None of the above

17. Find the volume of the solid bounded below by the  $xy$ -plane and above by the paraboloid  $z = 4 - x^2 - y^2$ .

- (a)  $\pi$       (b)  $2\pi$       (c)  $3\pi$       (d)  $4\pi$       (e) None of the above

18. Find the value of the line integral  $\int_{\gamma}(y+x)dx + xdy$  where  $\gamma(t) = (t^2e^{t^2}, t^4 - t)$ ,  $0 \leq t \leq 1$ .

- (a)  $\frac{e^2}{2}$       (b)  $e$       (c)  $\frac{e}{2} - \frac{1}{2}$       (d)  $e^2$       (e) None of the above

19. Let  $S$  be the oriented surface consisting of the part of the paraboloid  $z = 4 - x^2 - y^2$  above the  $xy$ -plane with outward normals. Let  $\vec{F}$  be the vector field

$$\vec{F}(x, y, z) = (-y + ze^x, x + y^4z, xz^3).$$

Use Stokes' Theorem to find the value of the surface integral  $\int \int_S (\nabla \times \vec{F}) \cdot d\vec{S}$ .

- (a)  $4\pi$       (b)  $2\pi$       (c)  $6\pi$       (d)  $8\pi$       (e) None of the above

20. Suppose the force at the point  $(x, y)$  is  $\vec{F}(x, y) = (-y, 2x)$ . Let  $\gamma$  be the curve consisting of the line segment from  $(1, 0)$  to  $(0, 1)$ . Find the work done by the force  $\vec{F}$  along the curve  $\gamma$ . [The magnitude of the force is in lbs. and distance is in ft.]

- (a)  $1 \text{ ft} \cdot \text{lb}$ .      (b)  $2 \text{ ft} \cdot \text{lb}$ .      (c)  $\frac{3}{2} \text{ ft} \cdot \text{lb}$ .      (d)  $\frac{5}{2} \text{ ft} \cdot \text{lb}$       (e) None of the above

21. Find the value of the line integral of the vector field

$$\vec{F}(x, y) = (3x^2y + e^x \sin(x))\vec{i} + (2x + x^3 + e^{y^2})\vec{j}$$

along the closed curve  $\vec{\gamma}(t) = \cos(t)\vec{i} + \sin(t)\vec{j}$ ,  $0 \leq t \leq 2\pi$ .

- (a) 0      (b)  $\pi$       (c)  $2\pi$       (d)  $-2\pi$       (e) None of the above

22. Let  $\vec{F}$  be the vector field  $\vec{F}(x, y, z) = y\vec{i} - x\vec{j} + \vec{k}$  and let  $S$  be the surface given by the part of the sphere  $x^2 + y^2 + z^2 = 1$  in the first octant [ $x \geq 0, y \geq 0, z \geq 0$ ] with normals pointing away from the origin. Evaluate the surface integral  $\int_S \vec{F} \cdot dS$ .

- (a) 0      (b)  $-\frac{\pi}{4}$       (c)  $\frac{\pi}{4}$       (d)  $\frac{\pi}{2}$       (e) None of the above

23. The quantity

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

- (a) exists and equals  $-1$    (b) exists and equals  $0$    (c) exists and equals  $1$   
(d) does not exist   (e) None of the above

24. Let  $S$  be the sphere given by  $x^2 + y^2 + z^2 = 1$  with outward normals. Let  $\vec{F}$  be the vector field

$$\vec{F}(x, y, z) = (3x, y, 4z).$$

Find the value of the surface integral  $\int \int_S \vec{F} \cdot dS$ .

- (a)  $16\pi$    (b)  $\frac{32\pi}{3}$    (c)  $\frac{16\pi}{3}$    (d)  $\frac{4\pi}{3}$    (e) None of the above