# MATH 311
# ABSTRACT ALGEBRA
# FINAL EXAM
# FALL 2006

**Name:**

No work $\Rightarrow$ No credit

10p. 1. Let $f(x) = x^3+1$, $g(x) = 2x^2+x+1$, $f, g \in \mathbb{Q}[x]$.
Perform the long division i.e., divide $f(x)$ by $g(x)$.

10p. 2. Compute $\gcd(35, 154)$ and find integers $a, b$, such that $d = 35a + 154b$.

15p. 3. Let $f(x) = x^3 - 8$, $g(x) = x^2 - x - 2$; $f, g \in \mathbb{Q}[x]$.
Compute $d(x) = \gcd(f(x), g(x))$ and find $a(x), b(x) \in \mathbb{Q}[x]$ such that $d(x) = f(x) \cdot a(x) + g(x) \cdot b(x)$

15p. 4. Decide whether the following polynomials are irreducible in $\mathbb{Q}[x]$.

(a) $f(x) = x^3 + x^2 + x + 1$
(b) $f(x) = x^4 + x^2 - 6$
(c) $f(x) = x^4 + x + 1$

15p. 5. Prove that a ring homomorphism $\phi: R \to S$ is one-to-one if and only if $\ker \phi = \{0\}$.

15p. 6. Find all ideals in the ring:

(a) $\mathbb{Z}$

(b) $\mathbb{Z}_6$

(c) $\mathbb{Z}_{16}$

20p. 7. Give the addition and multiplication table of

(a) $\mathbb{Z}_2[x]/\langle x^2+x\rangle$

(b) $\mathbb{Z}_2[x]/\langle x^2+1\rangle$

here $\langle\ \rangle$ stands for an ideal generated by:

15p. 8. Prove that any group of order $\leq 4$ is abelian.

15p. 9. Verify that if $\phi : G \longrightarrow G'$ is a group homomorphism, then $\text{image}(\phi) \subset G'$ is a subgroup.

20p. 10. What is the order of element:

ⓐ $5+\langle 6\rangle$ in the group $Z_{18}/\langle 6\rangle$

ⓑ $14+\langle 8\rangle$ in the group $Z_{24}/\langle 8\rangle$

15p. 11. The group $(Z_4 \times Z_{12})/\langle 2,2\rangle$ is isomorphic to:

ⓐ $Z_8$ ; ⓑ $Z_4 \times Z_2$ ; ⓒ $Z_2 \times Z_2 \times Z_2$

20p. 12 (A) If a group of order 35 acts on a set $S$ with 16 elements, show that the action must have a fixed point.
(B) List all possible numbers of fixed points of the action.

15p. 13. Let a finite group $G$ acts on a finite set $S$. Suppose that for each $s \in S$, $G_s = \{e\}$. Prove that
$N \cdot |G| = \#(S)$ ; here $N = \#$ (of distinct orbits in $S$)