

# ANSWER SHEET

# MATH 115 FINAL EXAM

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

FRIDAY, MAY 5, 2006 - 3:00 PM - 6:00 PM

1. Do NOT SEPARATE answer sheet from rest of test.
2. Work and CIRCLE the answer to each problem INSIDE this test.
3. Circle your answer a SECOND TIME on this page.
4. Blank answers are considered INCORRECT; no penalty for wrong answers
5. Do all 30 problems; each carries equal weight.

QUESTION      ANSWER

1.    a.    b.    c.    d.    e.

2.    a.    b.    c.    d.    e.

3.    a.    b.    c.    d.    e.

4.    a.    b.    c.    d.    e.

5.    a.    b.    c.    d.    e.

6.    a.    b.    c.    d.    e.

7.    a.    b.    c.    d.    e.

8.    a.    b.    c.    d.    e.

9.    a.    b.    c.    d.    e.

10.   a.    b.    c.    d.    e.

11.   a.    b.    c.    d.    e.

12.   a.    b.    c.    d.    e.

13.   a.    b.    c.    d.    e.

14.   a.    b.    c.    d.    e.

15.   a.    b.    c.    d.    e.

QUESTION      ANSWER

16.   a.    b.    c.    d.    e.

17.   a.    b.    c.    d.    e.

18.   a.    b.    c.    d.    e.

19.   a.    b.    c.    d.    e.

20.   a.    b.    c.    d.    e.

21.   a.    b.    c.    d.    e.

22.   a.    b.    c.    d.    e.

23.   a.    b.    c.    d.    e.

24.   a.    b.    c.    d.    e.

25.   a.    b.    c.    d.    e.

26.   a.    b.    c.    d.    e.

27.   a.    b.    c.    d.    e.

28.   a.    b.    c.    d.    e.

29.   a.    b.    c.    d.    e.

30.   a.    b.    c.    d.    e.

Math 115 FINAL EXAM — Spring 2006

Clearly mark one answer only. If two or more answer are marked, no credit will be given.  
No partial credit will be given if a wrong answer is marked.

Only non-graphic calculators are allowed. Good luck!

1. The distance from the point  $(1, 4)$  to the point  $(-2, 8)$  is:

- (a)  $\sqrt{17}$       (b)  $\sqrt{153}$       (c) 7      (d) 5      (e) none of the above

2. The equation of the line that goes through the points  $(1, 3)$  and  $(4, 9)$  is:

- (a)  $y = 2x + 1$       (b)  $y = -2x + 17$       (c)  $y = -2x + 1$       (d)  $y = 3x$   
(e) none of the above

3. The  $x$ -coordinate of a local minimum of  $f(x) = -2x^3 + 3x^2 + 12x + 5$  is:

- (a) 1      (b) -1      (c) 2      (d) 6      (e) none of the above

4. For the piecewise function  $f(x) = \begin{cases} \frac{2}{x}, & x < 2 \\ 3, & x = 2 \\ 3x - 5, & x > 2 \end{cases}$

- (a)  $\lim_{x \rightarrow 2} f(x) = 1$  and  $f'(2) = -\frac{1}{2}$       (b)  $\lim_{x \rightarrow 2} f(x) = 1$  and  $f'(2) = 3$   
(c)  $\lim_{x \rightarrow 2} f(x) = 3$  and  $f'(2) = -\frac{1}{2}$       (d)  $\lim_{x \rightarrow 2} f(x) = 3$  and  $f'(2) = 3$   
(e) none of the above

5. The equation of the line that is perpendicular to the line  $y = 2x + 6$  and goes through the point  $(4, -10)$  is:

- (a)  $y = 2x - 18$       (b)  $y = -2x - 2$       (c)  $y = \frac{1}{2}x - 12$       (d)  $y = -\frac{1}{2}x - 8$   
(e) none of the above

6. The functions  $f(x) = x^2$ ,  $g(x) = -3x + 2$ , and  $h(x) = x^3$  are defined for all real numbers  $x$ . Which functions are one-to-one?
- (a)  $f$  and  $g$       (b)  $g$  and  $h$       (c)  $f$  and  $h$       (d) only  $g$
- (e) none of the above
7. An interval on which the function  $f(x) = x^4 - 4x^3 + 400$  is concave down is:
- (a)  $(3, \infty)$       (b)  $(2, \infty)$       (c)  $(0, 2)$       (d)  $(-\infty, 0)$       (e) none of the above
8. For the piecewise function  $f(x) = \begin{cases} 3x + 2, & x < 1 \\ 6 - x, & x \geq 1 \end{cases}$
- (a)  $f'(1) = -1$       (b)  $f'_-(1) = 3$  and  $f'_+(1) = -1$
- (c)  $f'_-(1)$  does not exist and  $f'_+(1) = -1$       (d) neither  $f'_-(1)$  nor  $f'_+(1)$  exist
- (e) none of the above
9. The line  $L$  has slope 4 and contains the point  $(1, 2)$ . Which of the following points is also on  $L$ ?
- (a)  $(2, 8)$       (b)  $(4, 18)$       (c)  $(1, 4)$       (d)  $(5, 8)$       (e) none of the above
10. The functions  $f(x) = x^6 + 4x^2 - 3$ ,  $g(x) = \frac{1}{1 + x^2}$ , and  $h(x) = x^2 + 3x$  are defined for all real numbers  $x$ . Which functions are even?
- (a)  $f$  and  $g$       (b)  $g$  and  $h$       (c)  $f$  and  $h$       (d) only  $f$
- (e) none of the above
11. The function  $g(x) = x^4 - 24x^2 + 24$  has an inflection point at:
- (a)  $x = -1$       (b)  $x = -2$       (c)  $x = \sqrt{12}$       (d)  $x = -\sqrt{12}$
- (e) none of the above

12. The limit  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$  is equal to:

- (a)  $\sqrt{x}$       (b)  $(\sqrt{x})'$       (c)  $\frac{1}{\sqrt{x+h} + \sqrt{x}}$       (d) does not exist  
(e) none of the above

13. Given  $f(x) = \frac{x^2}{x^2 + 2x + 4}$ . Then  $f'(1)$  is equal to:

- (a)  $\frac{12}{49}$       (b)  $-\frac{12}{49}$       (c)  $\frac{10}{49}$       (d) 0      (e) none of the above

14. For which  $x$  is the function  $f(x) = x^3 + 3x^2 - 24x + 1$  decreasing?

- (a)  $x > 2$       (b)  $x < 0$       (c)  $-4 < x < 2$       (d)  $-1 < x < 2$   
(e) none of the above

15. Which of the following intervals contains a solution of the equation  $x^3 + x^2 - x = 2$ ?

- (a)  $(-2, -1)$       (b)  $(-1, 0)$       (c)  $(0, 1)$       (d)  $(1, 2)$       (e) none of the above

16. The  $x$ -coordinate of a point determined by the Mean Value Theorem for  $f(x) = x + \frac{9}{x}$  on the interval  $[1, 3]$  is:

- (a) 2      (b) -2      (c)  $\sqrt{3}$       (d)  $-\sqrt{3}$       (e) none of the above

17. Consider the following three limits: (i)  $\lim_{x \rightarrow -\infty} x^{-3}$ , (ii)  $\lim_{x \rightarrow -\infty} x^{\frac{1}{3}}$ , and

(iii)  $\lim_{x \rightarrow 1^+} \frac{x^2 - 3x + 2}{x^2 - 1}$ . Which of them are infinite?

- (a) (i) and (ii)      (b) (ii) and (iii)      (c) (i) and (iii)      (d) only (ii)  
(e) none of the above

18. Given  $f(x) = \frac{x^2}{x+1}$ . Which of the following is true?

- (a)  $x = 0$  is a local minimum point and  $x = -2$  is a local maximum point
- (b)  $x = 0$  is a local maximum point      (c)  $x = 2$  is a local maximum point
- (d)  $x = 4$  is a local minimum point      (e) none of the above

19. The limit  $\lim_{x \rightarrow 1} \frac{x^2 - 2x}{(x-1)^2}$  is equal to:

- (a)  $-2$       (b)  $-\infty$       (c)  $\infty$       (d)  $1$       (e) none of the above

20. The limit  $\lim_{x \rightarrow 1} \frac{\frac{2}{x+1} - 1}{x-1}$  is equal to:

- (a)  $\frac{1}{4}$       (b)  $-\frac{1}{2}$       (c)  $-1$       (d)  $1$       (e) none of the above

21. Given  $f(x) = x^{\frac{3}{2}}$  and  $g(x) = x^{\frac{7}{3}}$ . Which of the following is true?

- (a) the graph of  $g$  lies below the graph of  $f$  on the interval  $x \in (1, \infty)$
- (b) the graph of  $f$  lies below the graph of  $g$  on the interval  $x \in (0, 1)$
- (c) the graph of  $f$  lies below the graph of  $g$  on the interval  $x \in (0, \infty)$
- (d) the graph of  $g$  lies below the graph of  $f$  on the interval  $x \in (0, 1)$
- (e) none of the above

22. Given  $f(x) = \sqrt{x^4 - 2x + 1}$ . Then  $f'(0)$  is equal to:

- (a)  $-1$       (b)  $2$       (c)  $1$       (d)  $4$       (e) none of the above

23. The equation of the tangent line to  $x^3 + y^3 + y = 1$  at the point  $(-1, 1)$  is:

(a)  $y = \frac{1}{4}x + \frac{5}{4}$       (b)  $y = -\frac{3}{4}x + \frac{1}{4}$       (c)  $y = -x$       (d)  $y = 3x + 4$

(e) none of the above

24. The limit  $\lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 + x - 2}$  is equal to:

(a)  $\frac{1}{3}$       (b)  $\infty$       (c) 1      (d) does not exist      (e) none of the above

25. Given  $f(x) = (x^2 + 4)^{10}$ . Then its second derivative  $\frac{d^2f}{dx^2}$  is equal to:

(a)  $90(x^2+4)^8$       (b)  $(380x^2+80)(x^2+4)^8$       (c)  $360x^2(x^2+4)^8$       (d)  $20x(x^2+4)^9$

(e) none of the above

26. The global maximum point of the function  $f(x) = -x^2 + 4x$  on the interval  $3 \leq x \leq 4$  is:

(a)  $x = 3.5$       (b)  $x = 2$       (c)  $x = 3.75$       (d)  $x = 4$       (e) none of the above

27. A complete set of vertical and horizontal asymptotes of the function  $f(x) = \frac{2}{x+1} - \frac{1}{x+2}$  is:

(a)  $x = -1, x = -2, y = 1$       (b)  $x = -2, y = 0$

(c)  $x = -1, x = -2$       (d)  $x = -1, y = 0$       (e) none of the above

28. What are two real positive numbers  $x$  and  $y$  whose product is 10 and whose sum is as small as possible?

(a)  $x = 2, y = 5$       (b)  $x = 1, y = 10$       (c)  $x = \sqrt{10}, y = \sqrt{10}$

(d) the problem has no solution      (e) none of the above

29. Air is pumped into a ball at a rate of 10 cubic inches per second. How fast is the radius of the ball increasing at the instant that the ball has a radius of 2 inches?

(a) 2 in/sec      (b)  $\frac{2}{\pi}$  in/sec      (c)  $\frac{5}{8\pi}$  in/sec      (d)  $\frac{5}{2\pi}$  in/sec

(e) none of the above

30. If Katie walked at 3 miles per hour for 20 minutes and then sprinted at 10 miles an hour for 8 minutes, how fast would Dave have to walk or run to go the same distance as Katie did in the same time, while walking or running at a constant speed?

(a) 7 mph      (b) 6 mph      (c) 5 mph      (d) 4.5 mph      (e) none of the above