INSTRUCTOR ______________________ NAME Solutions

MATH 121 – FINAL EXAMINATION

Friday, May 5, 2006, 3 P.M.–6 P.M.

$f(-x) = \dfrac{e^{-\frac{1}{2}x}}{1+e^{-x}} \cdot \left(\dfrac{e^x}{e^x}\right) = \dfrac{e^{\frac{1}{2}x}}{e^x+1} = f(x)$

(1). The function $f(x) = \dfrac{e^{ax}}{1+e^x}$ is even when $a =$:

~~(a)~~ 0 (b) $\frac{1}{2}$ ~~(c)~~ 1 ~~(d)~~ -1 (e) none of these.

(2). The derivative of $\cos\left(e^x + \dfrac{1}{\sqrt{x}}\right)$ is: $-\left[e^x - \frac{1}{2}e^{-3/2}\right]\sin\left(e^x + \frac{1}{\sqrt{x}}\right)$

(a) $\left(e^x + \frac{1}{2}\sqrt{x}\right)\sin\left(e^x + \frac{1}{\sqrt{x}}\right)$; (b) $-\left(e^x - \frac{1}{2\sqrt[3]{x}}\right)\sin\left(e^x + \frac{1}{\sqrt{x}}\right)$;

(c) $\sin\left(e^x + \frac{1}{\sqrt{x}}\right)$; (d) $-\sin\left(e^x + \frac{1}{\sqrt{x}}\right)$; (e) none of these.

(3). When $a > 0$, the absolute minimum of $f(x) = x^2 + \dfrac{a}{x}$ on the interval $(0, \infty)$ is

(a) $\sqrt[3]{a}$ (b) $\sqrt{a}$ (c) $\sqrt[3]{a/2}$ (d) some other number

(e) there is no absolute minimum on that interval.

$3y^2y' + 2[2xy + x^2y'] - 6x = 0$

(4). At the point $(1,1)$ the curve $y^3 + 2x^2y - 3x^2 = 0$ has slope

(a) $\frac{2}{5}$ (b) $\frac{6}{5}$ (c) $\frac{6}{7}$ (d) some other number (e) does not exist.

(5). The average value of $f(x) = x^2$ on the interval $[0,2]$ is $\frac{1}{2-0}\int_0^2 x^2\,dx$

(a) 1 (b) $\frac{4}{3}$ (c) 2 (d) $\frac{8}{3}$ (e) none of these.

(6). You have 120 feet of fencing and want to enclose a rectangular area with one side on a long straight wall and fencing on the other three sides. The largest area you can enclose is:

(a) 800 sq.ft. (b) 1350 sq.ft. (c) 1600 sq.ft. (d) 2100 sq.ft. (e) none of these.

1800 sqft.

(7). The tangent line to the graph of $f(x) = (x+2)\,e^{-x}$ at the point $x = 0$ is:

(a) $y = 3x + 2$ (b) $y = 3x$ (c) $y = -x$

(d) $y = -x + 2$ (e) none of these.

(8). The horizontal asymptote of the graph of $f(x) = \dfrac{\sqrt{4x^2+1}}{2x-1}$ when $x \to \infty$ is

(a) $y = -1$ (b) $y = 1$ (c) $y = 2$ (d) $y =$ some other number

(e) this graph does not have a horizontal asymptote.

(9). The graph of $f(x) = \dfrac{\overset{(x+1)\cancel{(x-1)}\,(x+2)\cancel{(x-2)}}{\cancel{(x^2-1)(x^2-4)}}}{(x-1)^2\cancel{(x-2)}(x-3)}$ has how many vertical asymptotes

(a) 0 (b) 1 (c) 2 (d) 3 (e) some other number

(10). The position of a particle at time t is given by $s(t) = (t-1)(t-2)$. The particle is at rest when

(a) $t = 1$ only (b) $t = 2$ only (c) $t = 1$ and $t = 2$

(d) $t = \frac{3}{2}$; (e) none of these.

(11). $\lim_{x\to 1+} \dfrac{x^2-3x+2}{x^2-4x+3}$ equals $\lim_{x\to 1^+} \dfrac{(x-2)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} = \dfrac{1-2}{1-3} = \dfrac{-1}{-2} = \frac{1}{2}$

(a) 0 (b) $\frac{1}{2}$ (c) ∞ (d) $-\infty$ (e) none of these.

(12). $\lim_{n\to\infty} \dfrac{\pi}{2n} \sum_{i=1}^{i=n} \cos\left(\dfrac{i\pi}{2n}\right)$ equals $\int_0^{\pi/2} \cos x \, dx$

(a) 0 (b) 1 (c) ∞ (d) $-\infty$ (e) none of these.

(13). The absolute maximum of $f(x) = \dfrac{\ln(x)}{\sqrt{x}}$ occurs at

(a) $x = e$ (b) $x = 2e$ (c) $x = e^2$ (d) some other number

(e) this function does not have an absolute maximum.

(14). A solid of revolution is generated by revolving about the y-axis the first quadrant region below the curve $y = x\sqrt{1-x^3}$. Its volume is: $\int_0^1 (2\pi x)(x\sqrt{1-x^3})dx$

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{4\pi}{9}$ (d) 0 (e) some other number.

(15). A solid of revolution is generated by revolving about the x-axis the first quadrant region below the curve $y = \sqrt{1-x^3}$. Its volume is:

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{4\pi}{9}$ (d) $\dfrac{3\pi}{4}$ (e) some other number.

$V = \int_0^1 \pi \left(x\sqrt{1-x^3}\right)^2 dx$

(16). The area of the bounded region between the curves $y = x$ and $y = x^2 - x$ equals $\int_0^2 [x - (x^2 - x)]\, dx$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) 1 (e) some other number.

(17). $\int \frac{x^3\, dx}{1+x^4}$ equals:

(a) $\frac{x^4}{4} \tan^{-1}(x^2) + C$ (b) $\frac{1}{3} x^3 \tan^{-1}(x^2) + C$ (c) $\frac{1}{4} \ln|1 + x^4| + C$

(d) $\dfrac{\frac{x^4}{4}}{1 + \frac{x^5}{5}} + C;$ (e) none of these.

(18). $\int e^{3x}\, dx$ equals

(a) $e^{3x} + C$ (b) $3e^{3x} + C$ (c) $\frac{1}{3} e^{3x} + C$ (d) $\frac{\sqrt{\pi}}{17} e^{3x} + C$ (e) none of these.

(19). $\int \tan x \ln(\cos x)\, dx$ equals $-\frac{1}{2}(\ln(\cos x))^2 + C$

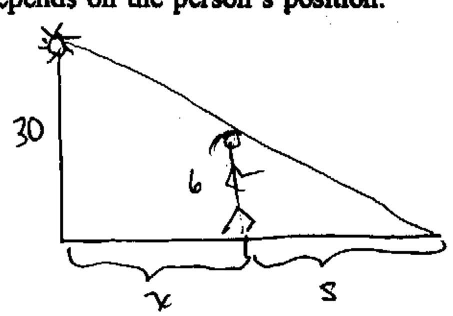
(a) $-\frac{1}{2}\left(\ln(\cos x)^2\right) + C;$ (b) $\ln(\cos x) + x \ln(\cos x)$ (c) $\sec^2 x \tan x + C$

(d) $-\ln(\cos x) + x \ln(\cos x)$ (e) none of these.

(20). A person 6 ft. tall is running away from a street light 30 ft high at the rate of 12 ft/sec. The length of the person's shadow is increasing at the rate of

(a) 2.4 ft/sec (b) 3 ft/sec (b) 14.4 ft/sec (b) 15 ft/sec (e) none of these.

(f) depends on the person's position.



$\frac{dx}{dt} = 12$

$\frac{ds}{dt} = ?$

$\frac{\overset{5}{36}}{6} = \frac{x+s}{s}$

$5s = x + s$

$4s = x$

$4\frac{ds}{dt} = \frac{dx}{dt}$

$\frac{ds}{dt} = \frac{1}{4}(12) = 3$