

Final Examination in Math 122 5 May 2006 3PM
MARK YOUR ANSWERS ON THIS SHEET

NAME: _____ INSTRUCTOR: _____

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| 21. | a | b | c | d | e | f | g | h |
| 22. | a | b | c | d | e | f | g | h |
| 23. | a | b | c | d | e | f | g | h |
| 24. | a | b | c | d | e | f | g | h |

Final Examination in Calculus 122 5 May 2006.
 MARK YOUR ANSWERS ON THE ANSWER SHEET (PRECEDING PAGE)
 AS WELL AS ON WHAT FOLLOWS.

- (1) The value of the Riemann sum for the integral of $f(x) = 4/x$ on the interval $[1, 2]$ using 4 sub-intervals and right endpoints in each subinterval is
- 533/210
 - 69/25
 - 319/105
 - integral diverges by p-test ($p = 1$) so there is no Riemann sum
 - $4 \ln x + C$, where C is a constant
 - $\ln 10$
 - $4 \ln 2$
 - None of the above.
- (2) The value of the integral

$$\int_2^4 x \ln x \, dx$$

is

- $4 \ln 2 + 4$
 - $3 - 7 \ln 2$
 - $10 \ln 2 + 1/4$
 - $-3 + 14 \ln 2$
 - $(x \ln(x))^2/2$
 - None of the above.
- (3) The value of the integral

$$\int_0^1 \frac{x+1}{x^2+5x+6} \, dx$$

is

- $\ln(27/32)$
 - $\ln 32 + \ln 27$
 - $5 \ln 2 - 3 \ln 3$
 - $(1 + \ln 2)/10$
 - The roots are complex so the integral is undefined
 - None of the above.
- (4) The volume of the solid of revolution obtained by rotating the portion of the curve $y = e^{-x}$ over $0 \leq x \leq 1$, about the line $y = 1$ is
- $\pi \int_0^1 (1 - e^{-x})^2 \, dx$
 - $\pi \int_0^1 (1 - e^{-x^2}) \, dx$
 - $\pi \int_0^1 (e^{-2x}) \, dx$
 - $\pi \int_0^1 (1 - e^{-2x}) \, dx$
 - The volume is infinite.
 - None of the above.

- (5) The partial fraction expansion of

$$\frac{x-1}{(x-2)(x-3)(x-4)(x^2+1)}$$

requires the determination of how many unknown coefficients?

- (a) 7
 (b) 1
 (c) 2
 (d) 3
 (e) 4
 (f) 5
 (g) 6
 (h) This expression has no partial fraction expansion
- (6) The length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ for $2 \leq x \leq 4$ is
- (a) 6.6931
 (b) $6 - \frac{1}{4}\ln 2$
 (c) $6 + \ln 2$
 (d) $8 - \frac{1}{4}\ln 2$
 (e) $6 + \frac{1}{4}\ln 2$
 (f) None of the above.

- (7) The integral

$$\int x \sin x \, dx$$

is

- (a) $-\frac{x^2}{2} \cdot \cos x + C$
 (b) $\cos x + C$
 (c) $-x \cos x + \sin x + C$
 (d) $x \cos x + \sin x + C$
 (e) None of the above.
- (8) The value of the integral

$$\int_0^{\infty} x e^{-3x} \, dx$$

is

- (a) 9
 (b) ∞ (diverges)
 (c) π
 (d) $\frac{1}{9}$
 (e) $\frac{1}{3}$
 (f) None of the above.
- (9) The value of the integral

$$\int_0^4 \sqrt{16-x^2} \, dx$$

is

- (a) $\pi/4$
 (b) 8π
 (c) 4π

- (d) $\arctan(4x) + C$
- (e) 16π
- (f) $4(\sqrt{2} + \sqrt{3})$
- (g) None of the above.

(10) The limit of

$$a_n = \left(\frac{n+1}{n}\right)^{8n}$$

as $n \rightarrow \infty$ is

- (a) 0
 - (b) 1^∞
 - (c) 1
 - (d) Does not exist
 - (e) e^8
 - (f) e^{48}
 - (g) ∞
 - (h) None of the above.
- (11) A cylindrical tank of radius 1 ft and height 4 ft is filled with a liquid weighing $w \frac{\text{lbs}}{\text{ft}^3}$. The work done in pumping all the liquid to the top of the tank is
- (a) $8\pi w$
 - (b) $24\pi w$
 - (c) $16\pi w$
 - (d) $4\pi w$
 - (e) None of the above.
- (12) The radius of convergence of the power series

$$f(x) = \sum_{k=0}^{\infty} 2 \frac{x^n}{n^2}$$

is

- (a) $1/2$
 - (b) 2
 - (c) Because of the coefficient 2, the series is not a power series, so has no radius of convergence
 - (d) ∞
 - (e) 1
 - (f) 0
 - (g) None of the above.
- (13) The radius of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{(-1)^n x^n}{(n+1)!}$$

is

- (a) 2
- (b) ∞
- (c) Because the series is alternating the radius oscillates between positive and negative values.

- (d) 0
 (e) 1
 (f) None of the above.
- (14) If the Taylor series of $f(x)$ and $g(x)$ about $a = 3$ have the same 5th-degree term, then
- (a) $\frac{f(x)}{5!} = \frac{g(x)}{5!}$ at $x = 3$
 (b) $f(x) - g(x)$ is a polynomial of degree 4 or lower.
 (c) $f(x) - g(x)$ is a polynomial of degree 5 or higher.
 (d) $f^{(3)}(5) = g^{(3)}(5)$
 (e) $f^{(5)}(3) = g^{(5)}(3)$
 (f) None of the above.
- (15) The absolute value of the error committed in making the approximation $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$ for $-1 < x < 1$ is
- (a) equal to $\frac{x^4}{4!}$
 (b) bigger than $\frac{1}{4}$
 (c) less than 10^{-6}
 (d) less than $1/8$
 (e) None of the above.
- (16) Let $y(x)$ be the solution of the initial value problem $\frac{dy}{dx} = y^3 - 9y$ with $0 < y(0) < 2$. What is $\lim_{x \rightarrow \infty} y(x)$?
- (a) ∞
 (b) -3
 (c) $y(0)$ because $y(0)$ is an equilibrium point
 (d) 3
 (e) 0
 (f) $-\infty$
 (g) None of the above.
- (17) Let $y(x)$ be the solution of the initial value problem

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

with $y(1) = 2$. The value of $y(2)$ is

- (a) 1
 (b) $\frac{17}{8}$
 (c) $\frac{35}{8}$
 (d) $\frac{23}{8}$
 (e) None of the above.
- (18) The Taylor series for $f(x) = \frac{1}{4+x}$ centered at $x = 0$ is
- (a) $\frac{1}{4} \sum_{n=0}^{\infty} x^n$
 (b) $\frac{1}{4} \sum_{n=0}^{\infty} (-1)^n x^n$
 (c) $\sum_{n=0}^{\infty} \frac{1}{4^n} x^n$
 (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^n$
 (e) None of the above.
- (19) Which of the following infinite series is equal to $\int_0^1 e^{-x^2} dx$?
- (a) $1 + 0 + 0 + 0 + \dots$
 (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)}$
 (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

- (d) $\sum_{n=0}^{\infty} \frac{1}{n^2(2n+1)}$
 (e) None of the above.
- (20) The mass $m(t)$ of a portion of a certain radioactive substance obeys the differential equation $dm/dt = -km$ where t is measured in years and k is a constant. It decreases from 2 grams to 1.9 grams in 50 years. How long will it take the substance to decrease from 2 grams to 1.8 grams? (Select answer closest to exact value.)
 (a) 99.01 years
 (b) 102.7 years
 (c) 100 years
 (d) 101.3 years
 (e) 103.4 years
- (21) The Maclaurin series (Taylor series centered at zero) for $f(x) = xe^{2x}$ is
 (a) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$
 (b) $\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$
 (c) $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$
 (d) $\sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n!}$
 (e) None of the above.
- (22) The series $\sum_{n=1}^{\infty} (\ln a)^n$ converges for exactly these a :
 (a) all real a
 (b) $e < a < e^2$
 (c) $\frac{1}{e} < a < e$
 (d) only for $a = 1$
 (e) None of the above.
- (23) Which one of these definite integrals is divergent?
 (a) $\int_0^{\infty} x^{10} e^{-x} dx$
 (b) $\int_1^{\infty} (x^3 + x^4)^{-1/2} dx$
 (c) $\int_0^{\infty} 1/(e^x + 2) dx$
 (d) $\int_0^{\infty} 1/(1 + \sqrt{x}) dx$
 (e) $\int_0^1 2/\sqrt{x} dx$
 (f) More than one of them
 (g) None of them
- (24) Which one of the following infinite series is divergent?
 (a) $\sum_1^{\infty} n(.88)^n$
 (b) $\sum_1^{\infty} n^2/(1 + n^3)$
 (c) $\sum_1^{\infty} \frac{\sin n}{n^2}$
 (d) $\sum_1^{\infty} \sin(1/n^2)$
 (e) More than one of these is divergent.
 (f) None of these is divergent.

Sign here if you wish to donate one point of your exam grade to a less well-prepared student: _____