Final Examination in Math 131 5 May 2006 3PM
MARK YOUR ANSWERS ON THIS SHEET

NAME:_____ INSTRUCTOR:_____

1.	A	B	C	D	E	F	G	H
2.	A	B	C	D	E	F	G	H
3.	A	B	C	D	E	F	G	H
4.	A	B	C	D	E	F	G	H
5.	A	B	C	D	E	F	G	H
6.	A	B	C	D	E	F	G	H
7.	A	B	C	D	E	F	G	H
8.	A	B	C	D	E	F	G	H
9.	A	B	C	D	E	F	G	H
10.	A	B	C	D	E	F	G	H
11.	A	B	C	D	E	F	G	H
12.	A	B	C	D	E	F	G	H
13.	A	B	C	D	E	F	G	H
14.	A	B	C	D	E	F	G	H
15.	A	B	C	D	E	F	G	H
16.	A	B	C	D	E	F	G	H
17.	A	B	C	D	E	F	G	H
18.	A	B	C	D	E	F	G	H
19.	A	B	C	D	E	F	G	H
20.	A	B	C	D	E	F	G	H
21.	A	B	C	D	E	F	G	H
22.	A	B	C	D	E	F	G	H
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Final Examination in Calculus 131 5 May 2006.
MARK YOUR ANSWERS ON THE ANSWER SHEET (PRECEDING PAGE)
AS WELL AS ON WHAT FOLLOWS.

(1) The derivative of $\cos(e^x + x^{-1/2})$ is
  (a) $\cos(e^x + x^{-1/2})\sin(e^x + x^{-1/2})$
  (b) $\cos(e^x + x^{-1/2})(e^x - .5x^{-1.5})$
  (c) $-\sin(e^x + x^{-1/2})(e^x - .5x^{-1.5})$
  (d) $\cos(e^x + x^{-1/2})(xe^{x-1} - .5x^{3/2})$
  (e) Derivative doesn't exist because of a division by zero
  (f) None of the above

(2) For what values of a does

$$\lim_{x\to 0} \frac{\sin x - x}{|x|^a}$$

exist? [Hint: Use a Taylor series.]
  (a) No values of a
  (b) All $a \leq 0$ and only those
  (c) All $a \leq 1$ and only those
  (d) All $a \leq 2$ and only those
  (e) All $a \leq 3$ and only those
  (f) Only $a = 1$
  (g) None of the above

(3) Suppose the tangent approximation to e^x at $x = 3$ is used to approximate e^x at $x = 2$. What approximation does it give?
  (a) $e^2 \approx 0$
  (b) $e^2 \approx 3.2$
  (c) $e^2 \approx 5.3$
  (d) $e^2 \approx 7.4$
  (e) $e^2 \approx 9.0$
  (f) None of the above

(4) Let a be some positive real number. Which of the following, if any, is the minimum value of the function

$$x^2 + \frac{2a^3}{x}$$

for $x > 0$?
  (a) The function approaches infinity as $x \downarrow 0$, so there's no minimum.
  (b) a
  (c) $2a^2$
  (d) $3a^3$
  (e) $a/2$
  (f) None of the above

(5) The value of the Riemann sum for the integral of $f(x) = 4/x$ on the interval $[1, 2]$ using 4 sub-intervals and using right endpoints in each subinterval is
  (a) 533/210
  (b) 69/25
  (c) 319/105
  (d) integral diverges by p-test ($p = 1$) so there is no Riemann sum
  (e) $4\ln x + C$, where C is a constant

(f) $\ln 10$
(g) $4 \ln 2$
(h) None of the above.

(6) The value of the integral

$$\int_0^1 \frac{x+1}{x^2+5x+6}\,dx$$

is
(a) $\ln(27/32)$
(b) $\ln 32 + \ln 27$
(c) $5\ln 2 - 3\ln 3$
(d) $(1+\ln 2)/10$
(e) The roots are complex so the integral is undefined
(f) the logarithm of something we haven't learned
(g) None of the above.

(7) The volume of the solid of revolution obtained by rotating the portion of the curve $y = e^{-x}$ over $0 \le x \le A$, about the line $y = 0$ is
(a) $\pi \int_0^A (1 - e^{-x})^2\,dx$
(b) $\pi \int_0^A \left(1 - e^{-x^2}\right)\,dx$
(c) $\frac{\pi}{2}(1 - e^{-2A})$
(d) πe^{-2A}
(e) $\pi \int_0^A \sqrt{1 + e^{-2x}}\,dx$
(f) The volume is infinite.
(g) None of the above.

(8) The partial fraction expansion of

$$\frac{x-1}{(x-2)(x-3)(x-4)(x^2+1)}$$

requires the determination of how many unknown coefficients?
(a) 7
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5
(g) 6
(h) This expression has no partial fraction expansion

(9) The length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ for $2 \le x \le 4$ is
(a) 6.6931
(b) $6 - \frac{1}{4}\ln 2$
(c) $6 + \ln 2$
(d) $8 - \frac{1}{4}\ln 2$
(e) $6 + \frac{1}{4}\ln 2$
(f) None of the above.

(10) The value of the integral

$$\int_0^\infty xe^{-3x}\,dx$$

is

(a) 9
(b) ∞ (diverges)
(c) π
(d) $\frac{1}{9}$
(e) $\frac{1}{3}$
(f) None of the above.

(11) The value of the integral

$$\int_0^4 \sqrt{16-x^2}\,dx$$

is

(a) $\pi/4$
(b) 8π
(c) 4π
(d) $\arctan(4x)+C$
(e) 16π
(f) $4(\sqrt{2}+\sqrt{3})$
(g) None of the above.

(12) The limit of

$$a_n = \left(\frac{n+1}{n}\right)^{8n}$$

as $n \to \infty$ is

(a) 0
(b) 1^∞
(c) 1
(d) Does not exist
(e) e^8
(f) e^{48}
(g) ∞
(h) None of the above.

(13) The radius of convergence of the power series

$$f(x) = \sum_{k=0}^{\infty} 2\frac{x^n}{n^2}$$

is

(a) 1/2
(b) 2
(c) Because of the coefficient 2, the series is not a power series, so has no radius of convergence
(d) ∞
(e) 1
(f) 0
(g) None of the above.

(14) The radius of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{(-1)^n x^n}{(n+1)!}$$

is

(a) 2
(b) ∞
(c) Because the series is alternating the radius oscillates between positive and negative values.
(d) 0
(e) 1
(f) None of the above.

(15) If the Taylor series of $f(x)$ and $g(x)$ about $a = 3$ have the same fifth degree term, then

(a) $\frac{f(x)}{5!} = \frac{g(x)}{5!}$ at $x = 3$
(b) $f(x) - g(x) =$ a polynomial of degree 4 or lower.
(c) $f(x) - g(x) =$ a polynomial of degree 5 or higher.
(d) $f^{(3)}(5) = g^{(3)}(5)$
(e) $f^{(5)}(3) = g^{(5)}(3)$
(f) None of the above.

(16) Let $y(x)$ be the solution of the initial value problem $\frac{dy}{dx} = y^3 - 9y$ with $0 < y(0) < 2$. What is $\lim_{x \to \infty} y(x)$?

(a) ∞
(b) -3
(c) $y(0)$ because $y(0)$ is an equilibrium point
(d) 3
(e) 0
(f) $-\infty$
(g) None of the above.

(17) Let $y(x)$ be the solution of the initial value problem

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

with $y(1) = 2$. The value of $y(2)$ is

(a) 1
(b) $\frac{17}{8}$
(c) $\frac{35}{8}$
(d) $\frac{23}{8}$
(e) None of the above.

(18) The Taylor series for $f(x) = \frac{1}{4+x}$ centered at $x = 0$ is

(a) $\frac{1}{4}\sum_{n=0}^{\infty} x^n$
(b) $\frac{1}{4}\sum_{n=0}^{\infty} (-1)^n x^n$
(c) $\sum_{n=0}^{\infty} \frac{1}{4^n} x^n$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^n$
(e) None of the above.

(19) The mass $m(t)$ of a portion of a certain radioactive substance obeys the differential equation $dm/dt = -km$ where t is measured in years and k is

a constant. It decreases from 2 grams to 1.9 grams in 50 years. How long will it take the substance to decrease from 2 grams to 1.8 grams? (Select answer closest to exact value.)

(a) 102.7 years
(b) 99.04 years
(c) 100 years
(d) 101.3 years
(e) 103.4 years

(20) The Maclaurin series (Taylor series centered at zero) for $f(x) = xe^{2x}$ is

(a) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$
(b) $\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$
(c) $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$
(d) $\sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n!}$
(e) None of the above.

(21) The series $\sum_{n=1}^{\infty} (\ln a)^n$ converges for exactly these a:

(a) all real a
(b) $e < a < e^2$
(c) $\frac{1}{e} < a < e$
(d) only for $a = 1$
(e) None of the above.

(22) Which one of these definite integrals is divergent?

(a) $\int_0^{\infty} x^{10} e^{-x}\, dx$
(b) $\int_1^{\infty} (x^3 + x^4 + 1)^{-1/2}\, dx$
(c) $\int_0^{\infty} 1/(e^x + 2)\, dx$
(d) $\int_0^{\infty} 1/(1 + \sqrt{x})\, dx$
(e) $\int_0^1 2/\sqrt{x}\, dx$
(f) More than one of them
(g) None of them

(23) Which one of the following infinite series is divergent?

(a) $\sum_1^{\infty} n(.88)^n$
(b) $\sum_1^{\infty} n^2/(1 + n^3)$
(c) $\sum_1^{\infty} \frac{\sin n}{n^2}$
(d) $\sum_1^{\infty} \sin(1/n^2)$
(e) More than one of these is divergent.
(f) None of these is divergent.

Sign here if you wish to donate one point of your exam grade to a less well-prepared student:_____