Mathematics 221 Spring 2006
Final Exam

(1) (a) Find a parametric representation of the line passing through the point $(\frac{8}{3}, 0, 1)$, which is parallel to the vector $\vec{v} = 2\vec{i} - 2\vec{j} + \vec{k}$.
(b) Find the normal to the plane $3x + 2y + 6z = 6$.
(c) Find the point where the line in part a) intersects the plane $3x + 2y + 6z = 6$.
(d) Find the distance from the point $(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$.

(2) If $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{w} = \vec{i} - \vec{j}$ and $\vec{u} = \vec{j} + 3\vec{k}$

(a) Compute the projection of the vector $\vec{v}$ onto the vector $\vec{w}$.

(b) Compute the cross product $\vec{u} \times \vec{w}$.

(c) Compute the angle between $\vec{v}$ and $\vec{u}$.

(d) Compute the volume of the parallelepiped spanned by these three vectors.

(3) Define the curve $\gamma(t) = (\cos 3t, \sin 3t, 2t)$ for $0 \le t \le 2$.

(a) Find the unit tangent vector $T(t)$ to the curve at an arbitrary time t.

(b) Find the length of the curve.

(c) Show that the curve lies on a circular cylinder of radius 1 centered at the origin.

(4) Find the directional derivative of $f(x, y) = x^2 + xy + y^2$ at the point $(1, -2)$ in the direction of $\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$.

(5) Find

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2},$$

or explain why the limit does not exist.

(6) Find the points on the curve $5x^2 + 2xy + 8y^2 = 5$ that are closest to the origin.

(7) Let D be the region in the plane that is inside the circle $x^2 + y^2 = 4$, outside the circle $x^2 + y^2 = 2x$ and to the right of the y-axis.

(a) Find the area of D.

(b) Evaluate

$$\iint_D y\,dA.$$

(c) Evaluate

$$\iint_D x\,dA.$$

(8) Let K be the solid that is below the sphere $x^2 + y^2 + z^2 = 16$ and above the cone $z = \sqrt{x^2 + y^2}$. (The solid K looks like an ice cream cone with a scoop of your favorite flavor on top.) Set up an iterated integral for, *but do not evaluate*,

$$\iiint_K (3y + z)dV$$

(a) in Cartesian coordinates.
(b) in cylindrical coordinates.
(c) in spherical coordinates.

(9) Let $\vec{F}(x,y,z) = y\cos(xy)\vec{\imath} + (x\cos(xy) + 2ye^z)\vec{\jmath} + y^2e^z\vec{k}$.

(a) Compute the divergence of $\vec{F}$.

(b) Compute the curl of $\vec{F}$.

(c) Does there exist a function $f(x,y,z)$ such that $\vec{F} = \nabla f$? If yes, find one such function. If not, explain why not.

(10) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F} = \nabla(x^3 e^{yz})$ along the helical curve $\vec{r}(t) = (\cos t, \sin t, t)$ where the parameter t satisfies $0 \leq t \leq \pi/2$.

(11) Find the flux of $\vec{F} = xz\vec{i} + x\vec{j} + -z^2\vec{k}$ inward through the parabolic cylinder $y = x^2$, where $0 \leq x \leq 1$, $0 \leq z \leq 4$.

(12) Compute the circulation of the field $\vec{F} = y^2\vec{i} - y\vec{j} + 3z^2\vec{k}$ around the ellipse where the plane $2x + 6y - 3z = 6$ intersects the cylinder $x^2 + y^2 = 1$, and the direction around the ellipse is counterclockwise as viewed from above.