

Math 111 Final Exam
December 11th, 2007

You have 4 hours to complete this exam. Please write your answers in the space provided and show all work. If additional space is needed, you may attach pages to the back of the exam. Please be sure to sign any attachments and clearly indicate the corresponding problem numbers.

1) Let X be a random variable with the following probability mass function:

x	1	2	3	4
$p(x)$	0.1	0.2	0.3	0.4

a) Find $P(X \geq 2)$.

b) Find $P(X \geq 2 | X < 4)$.

c) Find μ , the mean of X .

d) Find σ , the standard deviation of X .

2) 5% of the light bulbs produced in a factory are defective. A quality control inspector takes a random sample of 300 bulbs and counts the number of defective bulbs in the sample.

a) What is the exact distribution of X , the number of defective bulbs in the sample?

b) Find the approximate probability that X is less than 10, using the continuity correction.

3) On the Louisiana Lottery website "Frequently Asked Questions," a player posted the following comment: "*I purchased four scratch-offs where the odds were 1 in 4. I didn't win anything. Why is that?*"

a) If the probability of winning on any ticket is 0.25, find the probability of not winning at least once if four tickets are purchased.

b) Let X be the number of winners in a random sample of 50 of these scratch-off tickets. Find the mean and standard deviation of X .

c) By the Empirical Rule, would it be unusual to observe 25 winners in a random sample of 50 tickets?

4) A box contains five computer memory chips, two of which are defective. The chips are tested one at a time, recording D for a defective chip and G for a good one.

a) What is the probability that the first chip tested is defective?

b) If the first chip tested is found to be good, what is the probability that the second chip tested will also be good?

c) If two chips are simultaneously drawn from the box of five, what is the probability that at least one of them will not be defective?

d) If the 5 chips are placed in a row, how many distinct orderings of good and defective chips are possible?

5) 5 participants in a diet program lost an average of 4 pounds with sample standard deviation of 3 pounds.

a) Conduct a hypothesis test to determine whether or not the average weight loss was significantly greater than 0. Would you choose to reject the null hypothesis at significance level $\alpha = 0.01$?

b) The company sponsoring the diet program would really like to demonstrate that their method is successful. Given the observed sample mean $\bar{x} = 4$ and sample standard deviation $s = 3$, what sample size is required to observe statistically significant results at the $\alpha = 0.01$ level?

6) The number of chocolate chips in cookies made by the “Chock-O-Chips” company follows a Poisson distribution with mean $\mu = 5$ chips per cookie.

(a) What is the probability that a randomly selected cookie will contain 2 or fewer chips?

(b) What is the expected number of chips contained in a pack of 3 cookies?

(c) The rival company “Chips Galore” claims that their cookies contain more chocolate chips. A sample of 30 “Chips Galore” cookies yielded a sample mean of 6.7 chips with standard deviation 2.6 chips, while a sample of 30 “Chock-O-Chips” cookies had a sample mean of 5.2 chips with standard deviation of 2.3 chips. Construct a 95% confidence interval for the difference in means. Do the data support the company’s claim?

7) Ten measurements for carbon monoxide measurements at an oil refinery are as follows:
45 30 38 42 63 43 102 86 99 58

a) Draw a boxplot for the data.

b) Find the sample mean \bar{x} .

c) Find the sample standard deviation s .

d) Find a 95% confidence interval for the population mean μ .

8) Environmentalists are concerned about carbon monoxide levels greater than 80. Using the data provided in problem (7), conduct a hypothesis test to determine whether the population mean μ is greater than 80. Should the environmentalists be concerned? Use the $\alpha = 0.05$ significance level.

9) The Rotten Tomatoes website awards “Certified Fresh” status to films that receive good reviews from at least 75% of 40 or more critics. The Ridley Scott film “American Gangster” has received a 79% positive rating from 141 reviewers, and the Coen brothers film “No Country for Old Men” has received a 95% positive rating from 154 reviewers.

a) Construct a 99% confidence interval for the proportion of all reviewers who would favorably rate “American Gangster.” Can we conclude with 99% confidence that at least 75% of all reviewers would like this film?

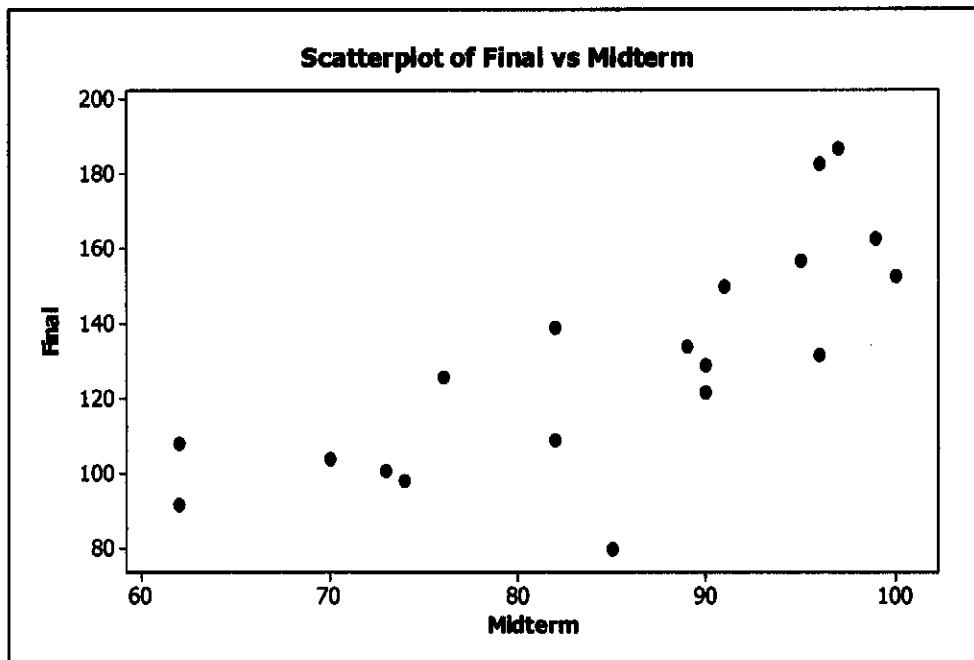
b) Construct a 99% confidence interval for the difference in proportions for the two films. Can we conclude with 99% confidence that reviewers prefer “No Country for Old Men” to “American Gangster”?

10) Also on the Rotten Tomatoes website, the film “Hitman” was given a positive rating by 11% of 63 reviewers.

a) Find a 95% confidence interval for the population proportion p .

b) If we assume that the true population proportion is close to 0.11, what sample size would be required to reduce the margin of error to 3%?

- 11) The following graph plots final exam grades against midterm grades for 19 students in a college Physics course, with summary statistics provided in the table below.



	Sample mean \bar{x}	Sample Standard Deviation s	Sample Correlation r
Midterm	84.68	12.14	0.758
Final	129.84	30.18	

- a) How would you describe the relationship between the midterm and final exam scores?
- b) Use the information provided in the table to derive the least-squares regression line for the data, and add the line to the graph above.
- c) What is the predicted final exam score for a student who has a score of 80 on the midterm exam?

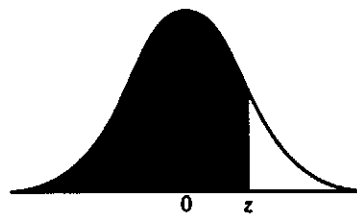


TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

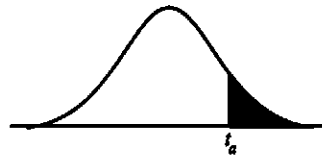


TABLE 4
Critical Values
of t

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
∞	1.282	1.645	1.960	2.326	2.576	∞

SOURCE: From "Table of Percentage Points of the t -Distribution," *Biometrika* 32 (1941):300. Reproduced by permission of the *Biometrika* Trustees.