

MATH 217, FALL SEMESTER 2007

Final

Name (print): _____
Signature: _____

INSTRUCTIONS

1. Calculators are NOT allowed to use.
2. Present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

QN	PTS
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____
11	_____
TOTAL	_____

1. True or False

(a) (2 points) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (\neg r \rightarrow \neg p)$

(b) (2 points) $((p \rightarrow q) \wedge (\neg q)) \rightarrow (\neg q)$

(c) (2 points) $((p \wedge q \rightarrow r) \wedge (\neg q)) \rightarrow (p \rightarrow r)$

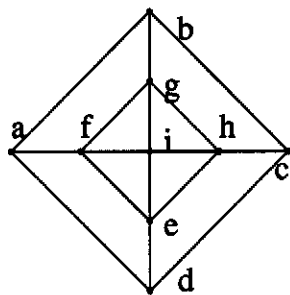
(d) (2 points) When a dice is thrown up, let E be the event that the number facing up is less than 4, F be the event that the number is greater than or equals to 4. Then two events are independent of each other.

2. (6 points) Evaluate

$$\sum_{i=0}^{100} \sum_{j=0}^{100} (i - j)$$

3. (6 points) In how many different ways can you pick 20 coins out of a piggy bank full of pennies, nickels, dimes and quarters?

4. Does the following graph have



(a) (2 points) an Euler path? Explain why.

(b) (2 points) an Euler circuit? Explain why.

(c) (2 points) a Hamilton path? Explain why.

(d) (2 points) a Hamilton circuit? Explain why.

(e) (2 points) What is its chromatic number? Explain.

5. (7 points) Some people get paid once every week. Prove that within a calendar year there is always a month that they get paid at least 3 times. Find the minimum number of consecutive months so that they are guaranteed to be paid at least 3 times in 6 different months.

6. (a) (4 points) How many words can be formed using all letters from the word "Louisiana"?

(b) (4 points) How many words can be formed using all letters from the word "Louisiana", if the i's cannot be next to each other?

(c) (4 points) How many words can be formed using all letters from the word "Louisiana", if the no two consecutive letters are the same?

7. Box A has 9 red balls and 2 black balls inside; box B has 5 red and 5 black. Joe reaches into the boxes, and randomly picks out balls.

(a) (4 points) What is the probability that Joe picks a red ball from box A?

(b) (4 points) What is the probability that Joe picks a red ball from box A, puts it back, then picks a black one from the same box?

(c) (4 points) What is the probability that Joe picks a red ball from box A, then picks a black one from the same box, without putting the first one back?

(d) (4 points) Now assume the boxes are unmarked. Joe randomly picks a box, get a ball from the box, and it is red. He picks another one from the same box without putting the first one back, and it is black. What is the probability the box is A?

8. (8 points) Find the number of integers between 1 and 1000, inclusive, that are not divisible by 5, 6, 8.

9. (9 points) A sequence is recursively defined as

$$f_{n+1} = 2f_n - f_{n-1} + 2, \quad f_0 = 0, \quad f_1 = 1.$$

Find a formula for the sequence, and prove your result.

10. (9 points) Prove that $\sqrt{5}$ is not rational.

11. (9 points) Given

$$a_{m,n} = \begin{cases} 1 & \text{if } m = 0 \text{ or } n = 0; \\ a_{m-1,n} + a_{m,n-1} & \text{otherwise,} \end{cases}$$

show that $a_{m,n} = \binom{m+n}{n}$.