1: The value of the following limit is

$$\lim_{x \to 0} \frac{(3+x)^{-1} - 3^{-1}}{x}$$

a. $-\frac{1}{9}$
b. $\frac{1}{3}$
c. $\frac{1}{9}$
d. Does not exist.
e. None of the above

2: The function $f(x) = x^{\frac{2}{3}}$ is differentiable on

a. $(-\infty, \infty)$
b. $[0, \infty)$
c. $(-\infty, 0) \cup (0, \infty)$
d. $(-\frac{2}{3}, \frac{2}{3})$
e. None of the above

3: Choose the function that is an *antiderivative* for $f(x) = 5x^3$.

a. $15x^4$
b. $\frac{5}{4}x^4$
c. $15x^2$
d. $\frac{5}{4}x^2$
e. None of the above

4: Which of the following polynomials satisfies the following conditions: it has a root at $x = 1$, its derivative vanish at $x = -1$, and its second derivative equals 8 at $x = 1$.

a. $-2 - 2x + 2x^2 + 2x^3$
b. $1 + 3x + 3x^2 + x^3$.
c. $-7 + 3x + 3x^2 + x^3$
d. $1 - x + x^2 + x^3$
e. None of the above

5: Consider the following polynomial $p(x) = ax^4 + 3x^2 + x + 1$ where $a < 0$, then

a. $\lim_{x\to\infty} p(x) = \infty$ and $\lim_{x\to-\infty} p(x) = -\infty$.
b. $\lim_{x\to\infty} p(x) = -\infty$ and $\lim_{x\to-\infty} p(x) = \infty$.
c. $\lim_{x\to\infty} p(x) = -\infty$ and $\lim_{x\to-\infty} p(x) = -\infty$.
d. $\lim_{x\to\infty} p(x) = \infty$ and $\lim_{x\to\infty} p(x) = \infty$.
e. None of the above

6: Suppose that you want to build a rectangular fence along the side of a river (note that only three side of fence are needed). Suppose that you have 1080 feet of fence material. Determine how to set up the fence so that the maximum amount of area is enclosed. Choose the appropriate answer.

a. It should be a 270 by 270 rectangular fence.
b. It should be a 270 by 540 rectangular fence.
c. It should be a 250 by 580 rectangular fence.
d. It should be a 580 by 580 rectangular fence.

7: The solution set for the inequality $\frac{2}{x+1} \geq 3$ is the interval

a. $(-\infty, \frac{1}{3}]$
b. $(-1, -\frac{1}{3}]$
c. $(-1, \frac{1}{3})$
d. $[-\frac{1}{3}, \infty)$
e. None of the above

8: How many solutions does the following system of equations have?

$$\begin{cases} x^2 - y = 0 \\ x + y = -3 \end{cases}$$

a. 0
b. 1
c. 2
d. 3
e. None of the above

9: The solution set to the inequality $|-3x+9| > 4$ is

a. $[\frac{5}{3}, \frac{13}{3}]$
b. $(-\infty, -\frac{5}{3}) \cup (\frac{5}{3}, \infty)$
c. $(-\infty, \frac{5}{3}) \cup (\frac{13}{3}, \infty)$
d. $(\frac{5}{3}, \frac{13}{3})$
e. None of the above

10: What is the domain of the function $f(x) = \frac{1}{\sqrt{x^2-1}}$?

a. $(-1, 1)$
b. $(\infty, -1] \cup [1, \infty)$
c. $[1, \infty)$
d. $(\infty, -1) \cup (1, \infty)$
e. None of the above

11: Let $(1, 2)$ and $(2, 5)$ be 2 ordered pairs in the Cartesian plane. The slope of a line perpendicular to the line passing through these two points is

a. $-\frac{7}{3}$
b. $-\frac{1}{3}$
c. $\frac{7}{3}$
d. 3
e. None of the above

12: The function $f(x) = |1 - 3x|$ is increasing on the interval

a. $(\frac{1}{3}, \infty)$
b. $(0, \infty)$
c. $(-\infty, \frac{1}{3})$
d. $(-\frac{1}{3}, \frac{1}{3})$
e. None of the above

13: Of the four functions $f_1(x) = x/(1+x^2)$, $f_2(x) = (x+1)^2-(x-1)^2$, $f_3(x) = |x-1|$, and $f_4(x) = x + x^2$, how many are odd?

a. 1
b. 2
c. 3
d. 4)
e. None of the above

14: How many of the functions in the preceding question (number 13) have an inverse?

a. 1
b. 2
c. 3
d. 4)
e. None of the above

15: If the following function

$$f(x) = \begin{cases} 2x + 1, x < 0 \\ g(x), x \geq 0 \end{cases}$$

is even, then
a. $g(x) = 2x + 1$
b. $g(x) = 2x - 1$
c. $g(x) = -2x + 1$
d. $g(x) = -2x - 1$
e. None of the above

16: The function $f(x) = \frac{x+2}{x+1}$, for all $x \neq -1$, has an inverse function $f^{-1}(x)$, and

a. $f^{-1}(x)) = \frac{2x-1}{x-1},\ x \neq 1$
b. $f^{-1}(x) = \frac{-x+1}{x-2},\ x \neq 2$
c. $f^{-1}(x) = \frac{x+1}{x+2},\ x \neq -2$
d. $f^{-1}(x) = \frac{-x+2}{x-1},\ x \neq 1$
e. None of the above

17: If $\lim_{x\to 1} f(x) = \frac{1}{2}$, then the value of the limit $\lim_{x\to 1} |x-1| f(x)$ is equal to

a. $1/2$
b. $-1/2$
c. 1
d. -1
e. None of the above

18: If $f(x)$ is a **continuous** function, and for all $x \neq 1$ we have $f(x) = \frac{x^2-2x+1}{x^2-6x+5}$, then the value of $f(1)$ is

a. 0
b. 1
c. -1
d. 5
e. None of the above

19: If the function

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, x < 1 \\ ax - 1, x \geq 1 \end{cases}$$

is continuous at $x = 1$, then the value of the constant a must be

a. 0
b. 1
c. 2
d. 3
e. None of the above

20: The function $f(x) = x^3 - 2x^2 - 3x$ is negative on the interval
a. $(-1, 0)$
b. $(1, 2)$
c. $(3, 4)$
d. $(5, 6)$
e. None of the above

21: Of the four functions $f_1(x) = |x|$, $f_2(x) = x|x|$, $f_3(x) = x^2|x|$, and $f_4(x) = x^3|x|$, how many are differentiable on their entire domain?
a. 1
b. 2
c. 3
d. 4
e. None of the above

22: If $f(x) = x^2$ is to have a tangent line at $x = a$ that is perpendicular to the tangent line which $f(x)$ has at $x = 1$, then the value of a must be
a. -1
b. $-1/2$
c. $-1/4$
d. $-1/8$
e. None of the above

23: If $f(x) = \sqrt{x-1}$ and $g(x) = 1/x$, then $f(g(1/4))$ is equal to
and $f_4(x) = x^3|x|$, how many are differentiable on their entire domain?
a. 2
b. 1
c. $\sqrt{3}$
d. $1/\sqrt{2}$
e. None of the above

24: For how many numbers a does the following limit exist?

$$\lim_{x \to a} \frac{x^3 + 2x^2 - x - 2}{x^2 - a^2}$$

a. 0
b. 1
c. 2
d. 3
e. None of the above

25: A spherical balloon with radius r inches has a volume $V(r) = \frac{4\pi}{3} r^3$. The function $f(r)$ which represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 5$ inches is a
a. linear function
b. quadratic polynomial
c. polynomial of degree 3
d. polynomial of degree 4
e. None of the above

26: The functions $f(x)$ and $g(x)$ are differentiable at $x = 0$. If $f(0) = 1$, $f'(0) = -1$, $g(0) = 2$, $g'(0) = 3$, and $F(x) = (x^2 + f(x))g(x)$, then the value of $F'(0)$ is equal to
a. -1
b. 0
c. 1
d. 2
e. None of the above

27: If $f(x)$ and $g(x)$ are as specified in the previous problem, and the new function $G(x)$ is given by $G(x) = \frac{f(x)}{f(x)+g(x)}$, then $G'(0)$ is equal to
a. 4/9
b. $-2/3$
c. 5/3
d. $-5/9$
e. None of the above

28: The function $f(x) = x^3 - 21x^2 + 80x^2 + 7$ satisfies three conditions of Rolle's Theorem. All the numbers c that satisfy Rolle's Theorem are:
a. $c_1 = 201 + \frac{\sqrt{7}}{3}, \quad c_2 = 201 - \frac{\sqrt{7}}{3}$
b. $c = 7 - \frac{\sqrt{201}}{3}$
c. $c_1 = 7 + \frac{\sqrt{201}}{3}, \quad c_2 = 7 - \frac{\sqrt{201}}{3}$
d. $c = 7 + \frac{\sqrt{201}}{3}$
e. $c = \frac{\sqrt{201}}{3}$

29: Find all the critical points of the function $f(x) = x^4(x-3)^3$.
a. $0,\ 2,\ \frac{12}{7}$
b. $0,\ 3,\ \frac{12}{11}$
c. $0,\ 3,\ \frac{12}{7}$
d. $0,\ 2,\ \frac{12}{11}$
e. $0,\ 2,\ \frac{7}{12}$

30: Find the point of the line $y = 4x + 8$ that is closest to the origin.
a. $(\frac{-32}{17}, \frac{10}{17})$
b. $(\frac{-34}{17}, \frac{9}{17})$
c. $(\frac{-32}{17}, \frac{8}{17})$
d. $(-2, \frac{8}{17})$
e. $(\frac{-31}{17}, \frac{8}{17})$

31: How many points of inflection are on the graph of the function $12x^3 + 14x^2 - 7x - 9$?
a. 3
b. 1
c. 4
d. 2
e. 5

32: For what values of c does the curve $f(x) = 5x^3 + cx^2 + 10x$ have both a local minimum and a local maximum?
a. $|c| > 15$
b. $|c| > \sqrt{150}$
c. $|c| > 1,500$
d. $|c| > \sqrt{30}$
e. $|c| > \sqrt{750}$

33: Using implicit differentiation, find the tangent line to the curve $x^3 - xy + y^3 = 1$ at the point $(1, 1)$.
a. $y = 3x - 2$
b. $y = x$
c. $y = x + 1$
d. $y = -x + 2$
e. None of the above

34: A baseball diamond is a square with each side having the length of 90 feet. A batter hits the ball and runs toward first base with a speed of 26 feet per second. At what rate is his distance from second base decreasing when he is halfway to first base? (For those of you unfamiliar with baseball, the batter starts at "home plate" which is one of the corners of the square. Moving counterclockwise around the square, first base is the next corner after home plate, and second base is the next corner after first base.)
a. $12/\sqrt{5}$ ft/sec
b. $90/\sqrt{5}$ ft/sec
c. $26/\sqrt{5}$ ft/sec
d. $\sqrt{5}/26$ ft/sec
e. None of the above

35: Find two positive numbers whose product is 144 and whose sum is a minimum.
a. 4, 36
b. 2, 72
c. 12, 12
d. 6, 24
e. None of the above

36: Find the most general antiderivative of the function $f(x) = 18x^2 - 14x + 9$.
a. $F(x) = 30x^5 - 28x^4 + 9x + C$
b. $F(x) = 6x^3 - 7x^2 + 9x + C$
c. $F(x) = 18x^3 - 14x^2 + 9x + C$
d. $F(x) = 36x - 14 + C$
e. None of the above

37: If $f(t) = \sqrt{4t+1}$, find $f''(2)$.
a. $-4/27$
b. $2/3$
c. $-2/3$
d. $4/27$
e. None of the above

38: What is the domain of the function $f(x) = \sqrt{9 - x^2}$?
a. $(-\infty, +\infty)$
b. $(-3, 3)$
c. $[-3, 3)$
d. $(-3, 3]$
e. None of the above

39: Evaluate the following infinite

$$\lim_{x \to \infty} \frac{7x^8 + 3x - 10}{6x^8 + 8x^4 + 5x^3 + 9}$$

a. $+\infty$
b. $-\infty$
c. 7
d. $7/6$
e. None of the above

40: The equation of the straight line through the two points $(3, 3)$ and $(-2, 5)$ is
a. $y = \frac{2}{5}x + \frac{21}{5}$
b. $y = -\frac{2}{5}x + \frac{21}{5}$
c. $y = \frac{2}{5}x + \frac{9}{5}$
d. $y = -\frac{5}{2}x + \frac{21}{2}$
e. None of the above