MATH 121 FINAL EXAM

Name: ____________________ **Section Number:** ________ **Instructor:** ___________

Tuesday, December 11, 2007 8:00 AM — 12:00 NOON

1. Do NOT SEPARATE answer page from rest of test.
2. Work and CIRCLE the answer to each problem INSIDE this test.
3. Circle your answers a SECOND time on this page.
4. One choice for one problem; no penalty for wrong answers.
5. Each problem carries equal weight: 5 points.
6. The following calculators are allowed: TI-82, 83 plus, 84 plus, 85 and 86; Calculators with symbolic manipulation capabilities are NOT allowed.

QUESTION	ANSWER
1.	a b c d e
2.	a b c d e
3.	a b c d e f
4.	a b c d e f
5.	a b c d e
6.	a b c d e
7.	a b c d
8.	a b c d e
9.	a b c d e
10.	a b c d e
11.	a b c d e
12.	True False
13.	a b c d e
14.	a b c d e
15.	a b c d e
16.	a b c d e
17.	a b c d e
18.	a b c d

QUESTION	ANSWER
19.	a b c d e
20.	a b c d e
21.	a b c d e
22.	a b c d e
23.	a b c d e
24.	a b c d e
25.	a b c d e
26.	a b c d e
27.	a b c d e
28.	a b c d
29.	a b c d
30.	a b c d e f
31.	a b c d

Mathematics 121 Final Exam 12/11/2007

1. What is the domain of the function

$$f(x) = \frac{1}{(1+x)(1-e^x)}$$

(a) All reals.

(b) All reals except the number -1.

(c) All reals except 0.

(d) All reals except 0 and -1.

(e) All reals except 1 and -1.

2. Simplify the expression $\tan(\cos^{-1}(x))$

(a) $\frac{\tan x}{\cos x}$.

(b) $\frac{\sqrt{1-x^2}}{x}$.

(c) $\frac{x}{\sqrt{1-x^2}}$

(d) x.

(e) $\tan\left(\frac{1}{\cos x}\right)$

3. If it exists as a finite number or infinity, find

$$\lim_{x \to 1^-} \frac{1}{x-1} - \frac{1}{|x-1|}.$$

(a) Does not exist.

(b) $\infty - \infty$.

(c) ∞

(d) $-\infty$.

(e) 0.

(f) 1.

4. If it exists as a finite number or infinity, find

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right).$$

(a) Does not exist.

(b) $\infty - \infty$.

(c) ∞.

(d) $-\infty$.

(e) 0.

(f) 1

5. Find

$$\lim_{x\to -2^+}\frac{x^2-4}{|x+2|}.$$

(a) 4.

(b) -4.

(c) 2.

(d) 0

(e) ∞

6. For what value of the constant c is the function f continuous on $(-\infty,\infty)$?

$$f(x)=\begin{cases} cx^3+3x & \text{if } x<-1 \\ cx^2 & \text{if } x\geq -1 \end{cases}$$

(a) There is no such c.

(b) 3.

(c) $-3/2$.

(d) Any c is good.

(e) $3x/(x^2-x^3)$.

7. Which of the following statements is true?

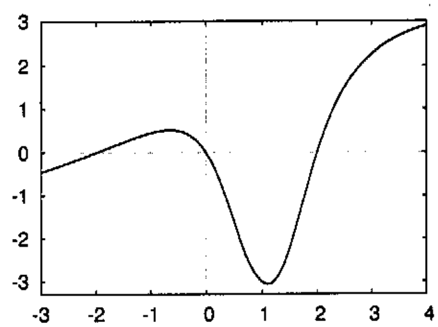
(a) The graph of $y = f(x)$ **can** intersect its asymptotes — both vertical and horizontal ones

(b) The graph of $y = f(x)$ **can** intersect its vertical asymptotes but **cannot** intersect its horizontal asymptotes

(c) The graph of $y = f(x)$ **can** intersect its horizontal asymptotes but **cannot** intersect its vertical asymptotes

(d) The graph of $y = f(x)$ **cannot** intersect its asymptotes — neither vertical nor horizontal ones

8. For the function g whose graph is given, which inequality is true?



(a) $g'(0) < 0 < g'(-1) < g'(3) < g'(2)$

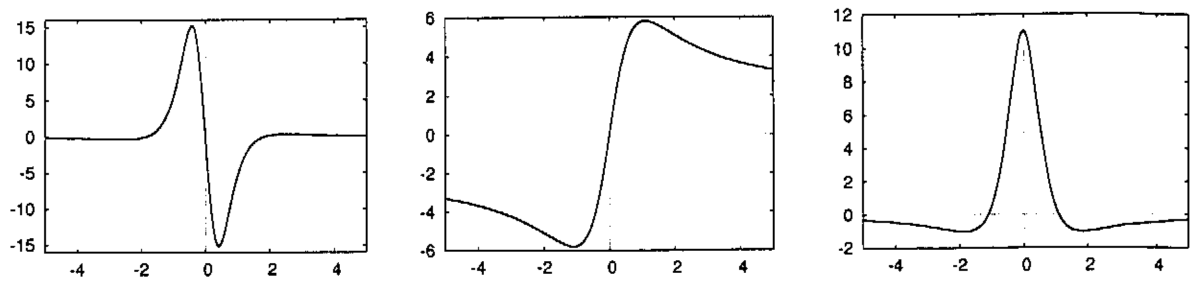
(b) $g'(-1) < 0 < g'(0) < g'(2) < g'(3)$

(c) $g'(0) < g'(-1) < 0 < g'(3) < g'(2)$

(d) $g'(0) < 0 < g'(-1) < g'(2) < g'(3)$

(e) none of the above

9. The figures show the graphs of f, its first and second derivatives in the following order (from left to right)



(a) $f, \ f', \ f''$
(b) $f', \ f, \ f''$
(c) $f'', \ f, \ f'$
(d) $f, \ f'', \ f'$
(e) none of the above

10. The graph of $f(x) = x^3 + 3x^2 + x + 3$ has a horizontal tangent at

(a) $x = -3$
(b) $x = -1 + \sqrt{\frac{3}{2}}$ and $x = -1 - \sqrt{\frac{3}{2}}$
(c) $x = -1$
(d) $x = -3 + \sqrt{\frac{2}{3}}$ and $x = -3 - \sqrt{\frac{2}{3}}$
(e) none of the above

11. The equation of the tangent line to the graph of $f(x) = \dfrac{2x}{x+1}$ at the point $(1, 1)$ is

(a) $y = \dfrac{2}{(x+1)^2}(x-1) + 1$
(b) $y = x$
(c) $y = \dfrac{x-1}{2}$
(d) $y = \dfrac{x+1}{2}$
(e) none of the above

12. Is the following statement **true** or **false**?

$$\frac{d}{dx}|x^2 + x| = |2x + 1|$$

True False

13. If $f(x) = g(x)^{h(x)}$, and $g(0) = 1, g'(0) = 2, h(0) = 3, h'(0) = 4$, evaluate $f'(0)$.

(a) 6
(b) 3
(c) e^6
(d) 64
(e) none of above

14. Given $f(x) = e^{x^2}$, find $f'(x)$?

(a) $x^2 e^{x^2-1}$
(b) $2x + e^{x^2}$
(c) $2xe^{x^2}$
(d) e^{x^2}
(e) none of above

15. Find the equation of the tangent line of the curve at $(e, 1)$

$$y = \ln(\frac{x}{y})$$

(a) $y - 1 = \frac{e+1}{e}(x - e)$
(b) $y - 1 = \frac{1}{e}(x - e)$
(c) $y - 1 = x - e$
(d) $y - 1 = \frac{1}{2e}(x - e)$
(e) none of above

16. The radius r of a sphere changes at a rate of $1/4$ m/s. Find the instantaneous rate of change of the volume of the sphere when $r = 2$.

(a) $8\pi/3$

(b) 16π

(c) 8π

(d) 4π

(e) none of above

17. A bacteria culture grows with constant relative growth rate. The number of bacteria grows from 100 to 2,000 in 6 hours. Find the number of bacteria after 12 hours.

(a) 40,000

(b) 20,000

(c) 12,000

(d) 4,000

(e) none of above

18. A man jogs along a square-shaped field at the speed of 6 miles per hour. Each side of the field is 1 mile long. If he started from a corner of the field, how fast is the distance changing from his current position to that corner after 15 minutes?

(a) 3.27 miles per hour

(b) 4.99 miles per hour

(c) -4.99 miles per hour

(d) 2.68 miles per hour

19. Given $f(1) = 1, f'(1) = 2, f(2) = 3, f'(2) = 4$ and $g(x) = f(2x)$, use linear approximation to estimate $g(1.1)$.

(a) 1.4.

(b) 3.8.

(c) 3.4.

(d) 1.2.

(e) none of above.

20. The function $f(x) = x^3 - 21x^2 + 80x + 7$ satisfies three conditions of Rolle's Theorem. All the numbers c that satisfy Rolle's Theorem are:

(a) $c_1 = 201 + \frac{\sqrt{7}}{3}, \quad c_2 = 201 - \frac{\sqrt{7}}{3}$

(b) $c = 7 - \frac{\sqrt{201}}{3}$

(c) $c_1 = 7 + \frac{\sqrt{201}}{3}, \quad c_2 = 7 - \frac{\sqrt{201}}{3}$

(d) $c = 7 + \frac{\sqrt{201}}{3}$

(e) $c = \frac{\sqrt{201}}{3}$

21. Find all the critical points of the function $f(x) = x^4(x-3)^3$.

(a) $0, \ 2, \ \frac{12}{7}$

(b) $0, \ 3, \ \frac{12}{11}$

(c) $0, \ 3, \ \frac{12}{7}$

(d) $0, \ 2, \ \frac{12}{11}$

(e) $0, \ 2, \ \frac{7}{12}$

22. Find the point of the line $y = 4x + 8$ that is closest to the origin.

(a) $(\frac{-32}{17}, \frac{10}{17})$

(b) $(\frac{-34}{17}, \frac{9}{17})$

(c) $(\frac{-32}{17}, \frac{8}{17})$

(d) $(-2, \frac{8}{17})$

(e) $(\frac{-31}{17}, \frac{8}{17})$

23. Find the limit.

$$\lim_{t \to 0} \frac{3^t - 2^t}{t}$$

(a) ∞

(b) 0.

(c) $\ln 2 - \ln 3$

(d) $\ln 3 - \ln 2$

(e) 1

24. How many points of inflection are on the graph of the function $12x^3 + 14x^2 - 7x - 9$?

(a) 3

(b) 1

(c) 4

(d) 2

(e) 5

25. For what values of c does the curve $f(x) = 5x^3 + cx^2 + 10x$ have both a local minimum and a local maximum?

(a) $|c| > 15$

(b) $|c| > \sqrt{150}$

(c) $|c| > 1,500$

(d) $|c| > \sqrt{30}$

(e) $|c| > \sqrt{750}$

26. Determine a definite integral which is equal to the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{i}{n})^{121}\frac{1}{n}$$

(a) $\int_1^2 x^{121}\,dx$

(b) $\int_0^1 x^{121}\,dx$

(c) $\int_{1+\frac{i}{n}}^{2} x^{121}\,dx$

(d) $\int_{1+\frac{1}{n}}^{n} x^{121}\,dx$

(e) $\int_1^{\infty}(1+x)^{121}\frac{1}{n}$

27. $\frac{d}{dx}\int_0^{1/x}\arctan t\,dt =$

(a) $\frac{-1}{x^2(1+x^2)}$

(b) $\arctan(\frac{1}{x})$

(c) $\frac{-\arctan t}{x^2}$

(d) $\frac{-\arctan(\frac{1}{x})}{x^2}$

(e) can't do it because we didn't learn how to integrate $\arctan t$.

28. Which of the following statements is NOT true:

(a) If $1 \le f(x) \le 2$ on the interval $[1,2]$, then $1 \le \int_1^2 f(x)\,dx \le 2$.

(b) Suppose the length of a rod is l and the linear density function of the rod is $\rho(x)$. Then the total mass of the rod is given by $\int_0^l \rho(x)\,dx$.

(c) If $f(x)$ is odd and continuous on the interval $[-1,1]$, then $\int_{-1}^1 f(x)\,dx = 0$.

(d) Suppose a particle is moving on the x-axis and its velocity function is $v(t)$. Then the total distance traveled by the particle in the time interval $[0,121]$ is equal to $\int_0^{121} v(t)\,dt$.

29. $\int(\frac{5+4x+3x^2}{x^2} + 2\sin x - 3e^x)\,dx =$

(a) $\frac{3(5x+2x^2+x^3)}{x^3} - 2\cos x - 3e^x + C$

(b) $\frac{-5}{x} + 4\frac{x^0}{0} + 3x - 2\cos x - 3e^x + C$

(c) $\frac{-5}{x} + 4\ln|x| + 3x + 2\sin x - 3e^x + C$

(d) $\frac{-5}{x} + 4\ln|x| + 3x - 2\cos x - 3e^x + C$

30. Which of the following is equal to $\int_1^2 3x^2\sqrt{x^3+1}\,dx$?

(a) $\int_1^2 3(u^2-1)^{2/3}u\,du$

(b) $\int_2^9 u^{1/2}\,du$

(c) $\int_1^2 u^{1/2}\,du$

(d) $\int_1^2 3\sqrt{x^{12}+x^4}\,dx$

(e) More than one of the above.

(f) None of the above.

31. Let a be a positive constant. Then $\int_0^a 2(x-a)e^{(x-a)^2}\,dx =$

(a) $1-e^{a^2}$

(b) $-a^2e^{a^2}$

(c) $2(x-a)(1-e^{a^2})$

(d) $-a^2(1-e^{a^2})$