Name _____

Calculus II

Final Exam

December 11, 2007

Score _____

Please do not separate the pages

Each problem carries a weight of 4 points

1) If $F(x) = \int_0^{x^2} f(t)dt$, and $F'(x) = 2x^7 \sin(x^2)$, then $f(x) =$

[ ] $\frac{x^6}{2}\sin(x)$ [ ] $x^6 \sin(x^2)$ [ ] $x^3 \sin(x)$ [ ] $2x^3 \sin(x^2)$ [ ] None of the above

2) $\int_1^2 x\sqrt{2-x}dx =$

[ ] $\frac{14}{15}$ [ ] $\frac{13}{12}$ [ ] $\frac{16}{17}$ [ ] $\frac{9}{10}$ [ ] None of the above

3) $\int_1^2 x^3 \ln x\, dx =$

[ ] $8\ln 2$ [ ] $(2\ln 2) - \frac{3}{5}$ [ ] $(4\ln 2) - \frac{15}{16}$ [ ] $(\ln 2) + \frac{23}{17}$ [ ] None of the above

4) $\int_0^{\frac{\pi}{2}} \cos^4 x\ dx =$

[ ] $\frac{3\pi}{4}$ [ ] $\frac{3\pi}{16}$ [ ] $\frac{2\pi}{15}$ [ ] $\frac{2\pi}{9}$ [ ] None of the above

5) If $F'(x) = \frac{1}{x^2\sqrt{4-x^2}}$ for $0 \prec x \prec 2$, and $F(2) = 0$, then $F(1) =$

[ ] $\frac{2}{\sqrt{3}}$ [ ] $\frac{\sqrt{3}}{4}$ [ ] $-\frac{2}{\sqrt{3}}$ [ ] $-\frac{\sqrt{3}}{4}$ [ ] None of the above

6) Let f be the solution of the initial value problem $y' - 2ty = t$, $y(0) = 0$. Then $f(1) =$

[ ] e [ ] $1 + e$ [ ] $\frac{e-1}{2}$ [ ] $e^4 - 1$ [ ] None of the above

7) The solutions to the differential equation $x + 2yy' = 0$ are

[ ] parabolas [ ] ellipses [ ] hyperbolas [ ] parallel lines

[ ] None of the above

8) Consider the initial value problem $\frac{dy}{dx} = x + y^2, \qquad y(0) = 1.$ The solution to this problem is an increasing function. Using Euler's method with step size $h = 0.1$,what is the smallest number of steps you need to reach at least 1.5 for the approximate value of the function?

[ ] 2 [ ] 3 [ ] 4 [ ] 5 [ ] None of the above

9) The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 4 \cdot 7 \cdot \cdots (3n-2)} x^n$ equals

[ ] $\frac{1}{3}$ [ ] $\frac{2}{3}$ [ ] 1 [ ] 3 [ ] None of the above

10) Which is the largest interval such that the power series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ converges for all numbers in that interval

[ ] [-1,1] [ ] (-1,1) [ ] (-1,1] [ ] [-1,1) [ ] None of the above

11) The power series $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$ converges absolutely in the interval (-1,1) and its limit equals

[ ] $\arctan(x)$ [ ] $\ln(1/(1-x))$ [ ] $\ln\sqrt{\frac{1+x}{1-x}}$ [ ] $\frac{1}{1+x^2}$ [ ] None of the above

12) Using the Taylor polynomial of degree 2 of $f(x) = \sqrt{x}$ at $a = 25$ to approximate $\sqrt{26}$, yields the number

[ ] 5.101 [ ] 5.100 [ ] 5.098 [ ] 5.099 [ ] None of the above

13) Expressing the number $\int_0^1 e^{x^2} dx$ as the limit of an infinite series yields

[ ] $\sum_{n=0}^{\infty} \frac{1}{n!}$ [ ] $\sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$ [ ] $\sum_{n=0}^{\infty} \frac{1}{(2n+1)n!}$ [ ] $\sum_{n=0}^{\infty} \frac{1}{(n+1)n!}$

[ ] None of the above

14) The smallest degree of a Taylor polynomial that is needed to approximate $\sin(x)$ with an error less than 0.001 for all x in the interval $[-\pi/2, \pi/2]$ is

[ ] 1 [ ] 2 [ ] 3 [ ] 4 [ ] None of the above

15) The length of the curve $y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$ between $x = 1$ and $x = 3$ is

[ ] $\frac{28}{3}$ [ ] $\frac{53}{6}$ [ ] $\frac{35}{4}$ [ ] $\frac{44}{5}$ [ ] None of the above

16) The indefinite integral $\int \frac{dx}{(x^2 + 2x + 7)^3}$ involves a term of the form $c \cdot \arctan((x + a)/b)$. The value of b is

[ ] 1 [ ] $\sqrt{2}$ [ ] $\sqrt{6}$ [ ] $\sqrt{7}$ [ ] None of the above

17) If $\int \frac{3x^2 + 4x + 2}{x(x+1)^3} dx$ is written in the form $\int \frac{A}{x} dx + \int \frac{B}{x+1} dx + \int \frac{C}{(x+1)^2} dx + \int \frac{D}{(x+1)^3} dx$, then D equal

[ ] 0 [ ] -1 [ ] $-\frac{1}{2}$ [ ] $\frac{1}{2}$ [ ] None of the above

18) The improper integral $\int_1^\infty f(x)dx$ converges if $f(x) =$

[ ] $\sin x$ [ ] $\frac{x-1}{\sqrt{x^3+1}}$ [ ] $\frac{x}{1+e^x}$ [ ] $\frac{2+\cos x}{x}$ [ ] None of the above

19) The series $\sum_{n=1}^{\infty} a_n$ converges if $a_n =$

[ ] $\frac{\sqrt{n^2+2}}{n^2+1}$ [ ] $\frac{e^n}{n^e}$ [ ] $\tan\left(\frac{1}{n^2}\right)$ [ ] $\frac{1}{n \cdot \ln(n)}$ [ ] None of the above

20) The volume of the solid obtained by revolving the region bounded by $x = (y-1)^2$ and $x = y+1$ about the x-axis equals

[ ] $\frac{27\pi}{2}$ [ ] 14π [ ] $\frac{25\pi}{2}$ [ ] 12π [ ] None of the above

21) The number of values a for which the series $\sum_{n=1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$ converges is

[ ] 0 [ ] 1 [ ] 2 [ ] infinite [ ] None of the above

22) The area of the region common to the circles $r = \cos\theta$ and $r = \sin\theta$ is

[ ] $\frac{\pi+1}{4}$ [ ] $\frac{\pi-1}{4}$ [ ] $\frac{2\pi+1}{8}$ [ ] $\frac{\pi-2}{8}$ [ ] None of the above

23) The natural length of a spring is 10cm. If it requires 2 ergs of work to stretch the spring to a total length of 18 cm, how much work (in ergs) will be performed in stretching the spring to a total length of 20 cm?

[ ] 3.125 [ ] 2.874 [ ] 3.317 [ ] 2.572 [ ] None of the above

24) The curve given by the parametric equations $x = \sin\pi t$, $y = \cos 2\pi t$ is a

[ ] hyperbola [ ] ellipse [ ] parabola [ ] cardoid [ ] None of the above

25) The area of the surface generated by revolving about the x-axis the arc of the curve $y = x^3$ on [0,1] is approximately

[ ] 3.5631 [ ] 2.896 [ ] 3.253 [ ] 2.497 [ ] None of the above