MATH 111, Final exam, 4/5/2007

Name:

Section: Lab time:

1	a	b	c	d	e	13	a	b	c	d	e
2	a	b	c	d	e	14	a	b	c	d	e
3	a	b	c	d	e	15	a	b	c	d	e
4	a	b	c	d	e	16	a	b	c	d	e
5	a	b	c	d	e	17	a	b	c	d	e
6	a	b	c	d	e	18	a	b	c	d	e
7	a	b	c	d	e	19	a	b	c	d	e
8	a	b	c	d	e	20	a	b	c	d	e
9	a	b	c	d	e	21	a	b	c	d	e
10	a	b	c	d	e	22	a	b	c	d	e
11	a	b	c	d	e	23	a	b	c	d	e
12	a	b	c	d	e	24	a	b	c	d	e

1. You roll die. What is the probability that you get an outcome that is even or less than 3?

(a) 2/6
(b) 3/6
(c) 4/6
(d) 5/6
(e) 1

2. A loaded six-sided die has the following properties: (a) all odd outcomes are equally likely, (b) all even outcomes are equally likely, and (c) you are twice as likely to get an even number as you are to get an odd number. If you roll the die, what is the probability that you get 6?

(a) 1/36
(b) 1/9
(c) 2/9
(d) 1/6
(e) 1/2

3. Let A and B be independent events such that $P(A) = 0.5$ and $P(B) = 0.5$. What is $P(A \cup B)$?

(a) 0
(b) 0.25
(c) 0.5
(d) 0.75
(e) 1

4. Roll two dice and consider the following three events:

A: the sum equals nine, B: at least one die shows 1, C: one die shows 3

Which pairs of events are mutually exclusive?

(a) Only (A, B)
(b) All pairs
(c) None of the pairs
(d) (A, B) and (B, C)
(e) Only (B, C)

5. You are dealt a poker hand. Below is an expression for the probability to get three of a kind but one factor X is missing. What is X?

$$P(\text{three of a kind}) = \frac{13 \cdot \binom{4}{3} \cdot X \cdot 4 \cdot 4}{\binom{52}{5}}$$

(a) $X = 48$
(b) $X = \binom{4}{2}$
(c) $X = \binom{48}{2}$
(d) $X = 12$
(e) $X = \binom{12}{2}$

6. In American roulette there are the 38 numbers 00 and 0-36. If you wager \$1 on a straight bet (bet on one single number), the payout is \$35 if you win which makes the expected gain $-2/38$. In a *split bet*, you bet on two numbers and if any of these two come up, you win. If you wager \$1 on a split bet, what should the payout be in order to give the same expected gain as with a straight bet?

(a) 8
(b) 11
(c) 15
(d) 17
(e) 17.50

7. You have estimated that there is a 5% chance that something is wrong with your car each time you start it. If something is wrong, there is a 75% chance that the "check engine" light comes on. If nothing is wrong, there is still a 10% chance that the light comes on in error. What is the probability that the light comes on when you start your car?

(a) 5%
(b) 10%
(c) 13%
(d) 50%
(e) 75%

8. You test for a disease that one in a thousand people have. The test comes back either positive or negative. If you have the disease, the test will be positive with 99% probability and if you do not have the disease, the test will be negative with 95% probability. If you test positive, what is the probability that you have the disease?

(a) 0.001
(b) 0.02
(c) 0.05
(d) 0.95
(e) 0.99

9. You choose a password by choosing four letters followed by three digits. How many possible such passwords are there if there are no restrictions on how many times each letter or digit can be used.

(a) $26^4 \cdot 10^3$
(b) $\binom{26}{4} \cdot \binom{10}{3}$
(c) $26! \cdot 10!$
(d) $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 \cdot 8$
(e) $26! \cdot 10!/7!$

10. A class consists of 20 girls and 10 boys. If you choose 5 students at random, what is the probability that it contains exactly 3 girls?

(a) 0.008
(b) 0.15
(c) 0.33
(d) 0.36
(e) 0.67

11. You roll a fair die 600 times and let x denote the number of 6s. What is $P(90 < x < 110)$?

(a) 0.698
(b) 0.702
(c) 0.726
(d) 0.746
(e) 0.750

12. There are currently 16 women serving in the U.S. Senate; 5 Republicans and 11 Democrats. The Senate as a whole has 49 Republicans and 51 Democrats (including 2 Independents who caucus with the Democrats). A Senator is chosen at random and the events F: female and D: Democrat are considered. What is the conditional probability $P(D|F)$?

(a) 0.51
(b) 0.11
(c) 0.22
(d) 0.69
(e) 0.16

13. The number of misprints per page in a newspaper is a random variable x that can take the values 0, 1, 2, or 3 with probabilities $0.50, 0.05, 0.40$, and 0.05, respectively. What is the expected value of x, $E[x]$?

(a) 0
(b) 0.5
(c) 1
(d) 1.5
(e) 2

14. You flip a fair coin 100 times. What is the probability that you get 50 heads and 50 tails?

(a) 0.08
(b) 0.54
(c) 0.5
(d) 0.25
(e) 1

15. IQ scores in a population follow a normal distribution with mean 100 and standard deviation 15. In a sample of 25 people, the average IQ $\bar{x}$ is measured. Then $\bar{x}$ has a normal distribution with

(a) mean 100 and standard deviation 15
(b) mean 100 and standard deviation 3
(c) mean 100 and standard deviation 0.6
(d) mean $2,500$ and standard deviation 75
(e) mean $2,500$ and standard deviation 375

16. You bid on an object at a silent auction. You estimate that the maximum bid from others is a random variable x that is uniform between 70 and 130 dollars. How much do you need to bid to have a 95% chance to win the bidding?

(a) 95
(b) 73
(c) 125
(d) 127
(e) 130

17. Suppose that IQ scores in a population follow a normal distribution with mean 100 and standard deviation 15. What is the probability that a randomly chosen individual has an IQ score below 75?

(a) 0.7%
(b) 4.8%
(c) 20.0%
(d) 75%
(e) 95.2%

18. In the previous problem, find the 99th percentile, that is, the IQ score that has 99% of the population below it and 1% above.

(a) 65
(b) 115
(c) 130
(d) 135
(e) 145

19. In an April 2007 opinion poll of 1,007 individuals, 363 said that they approved of President Bush. What is the 99% confidence interval for the percentage of supporters in the population?

(a) (32, 40)
(b) (33, 39)
(c) (31, 41)
(d) (30, 42)
(e) (29, 43)

20. In an opinion poll regarding an unknown proportion p, the margin of error in a 95% confidence interval equals $1.96\sqrt{\hat{p}(1-\hat{p})/n}$. If we instead want confidence level 90%, what does 1.96 need to be replaced by?

(a) 1.64
(b) 2.00
(c) 2.32
(d) 2.58
(e) 3.09

21. The support for a politician is measured in two consecutive opinion polls. The first poll gave support from 672 of 1,200 individuals and the second poll gave support from 711 of 1,250 individuals. What is the 95% confidence interval for the increase in support between the two polls (in percentage points)?

(a) (−4.2, 6.0)
(b) (−3.0, 4.8)
(c) (−2.4, 4.2)
(d) (1.6, 9.4)
(e) (−4.8, 3.0)

22. The output voltage of a generator was measured once a day for five days and gave the following data: 213, 215, 220, 221, 224. What is the 95% confidence interval for the unknown mean voltage μ? You can assume that the voltage follows a normal distribution.

(a) $(215, 223)$
(b) $(213, 224)$
(c) $(212, 226)$
(d) $(209, 228)$
(e) $(214, 223)$

23. Blood pressures in two populations are compared. A sample of 23 individuals from the first population gave an average (systolic) blood pressure of 150 with a sample standard deviation of 15. In the second population, 15 individuals were tested and gave an average of 130 and a sample standard deviation of 12. Suppose that blood pressures in the two populations follow normal distributions with unknown means and the same unknown standard deviation σ. What is the 90% confidence interval for the difference between the means?

(a) $(9, 31)$
(b) $(11, 29)$
(c) $(12, 28)$
(d) $(13, 27)$
(e) $(7, 33)$

24. You want to prove that striped bass in a certain river has a mean weight that exceeds 10 pounds. To support your claim, you catch and weigh 12 bass, giving a sample mean of 11.4 and a sample standard deviation of 1.9. Weights follow a normal distribution. What is the p-value of the relevant hypothesis test?

(a) 0.013
(b) 0.027
(c) 0.987
(d) 0.005
(e) 0.011