

Name _____

Math 115

Long Calculus I

Final Exam

May 4, 2007

Score _____

Please do not separate the pages

Each problem carries a weight of 4 points

1) At least how many intervals do you need to describe the solution set of the inequality $|x^2 + 4x - 1| \geq 4$?

1 2 3 4 None of the above

2) The line passing through the point $(-1, 3)$ which is parallel to $y = 3x - 2$ intersects the y -axis at $y =$

3 4 5 6 None of the above

3) The function $f(x) = \frac{1}{x+2}$, $x \neq -2$, has an inverse $g(x) =$

$\frac{1}{x-2}$, $x \neq 2$ $x+2$ $\frac{x}{x+2}$, $x \neq -2$ $\frac{1-2x}{x}$, $x \neq 0$

None of the above

4) If $f(x) = x - 1$, $g(x) = x^2$, $h(x) = \sqrt{x}$, then $h \circ g \circ f \circ g(x) =$

$(\sqrt{x}-1)^4$ $(\sqrt{x}-1)^2$ $\sqrt{x^2-1}$ $|x^2-1|$ None of the above

5) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right) =$

$\frac{3}{7}$ $\frac{5}{9}$ $\frac{5}{11}$ $\frac{3}{5}$ None of the above

6) For how many numbers c does $\lim_{x \rightarrow 1} \frac{x^2 + 3x + c}{x-1}$ exist?

0 1 2 4 None of the above

7) For how many numbers c does $\lim_{x \rightarrow 1^+} \frac{|x+c||x-c|}{x^2-1}$ exist?

0 1 2 4 None of the above

8) If $\lim_{x \rightarrow -2} f(x) = 4$ and $\lim_{x \rightarrow 2} g(x) = -4$, then $\lim_{x \rightarrow -2} g(\sqrt{f(x)}) =$

-4 -2 2 4 None of the above

9) $\lim_{x \rightarrow 0} \frac{(x+a)^2 - a^2}{x} =$

a -a 2a -2a None of the above

10) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1+a^2} - \sqrt{1+a^2}}{x} =$

$\frac{1}{2\sqrt{1+a^2}}$ $\frac{1}{1+\sqrt{1+a^2}}$ $\sqrt{1+a^2}$ $2\sqrt{1+a^2}$ None of the above

11) The function $f(x) = \begin{cases} \frac{x^4-1}{x^2-1} & \text{if } |x| \neq 1 \\ c & \text{if } |x| = 1 \end{cases}$ is continuous if $c =$

0 1 2 4 None of the above

12) For how many numbers c is the function $f(x) = \begin{cases} c^2x^2 + 1 & \text{if } x \geq 1 \\ 2cx & \text{if } x < 1 \end{cases}$ continuous?

0 1 2 3 None of the above

13) If $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$, then $f(x) =$

\sqrt{x} $-\sqrt{x}$ $\frac{1}{\sqrt{x}}$ $-\frac{1}{\sqrt{x}}$ None of the above

14) The graph of the function $f(x) = (x+1)^p$ has exactly one tangent line with slope -1 if $p =$

0 -1 -2 -3 None of the above

15) If $y = f(x)$ is a function such $f(2+h) - f(2) = 3h^2 + 5h$, then $f'(2) =$

2 3 4 5 None of the above

16) The line which is tangent to the graph of $y = \frac{1}{x}$ in the first quadrant, and which has the slope -1, intersects the y -axis at $y =$

0 1 2 3 None of the above

17) The function $f(x) = x^{\frac{1}{3}}g(x)$ is **not** differentiable at $x = 0$ if $g(x) =$

$x^{\frac{3}{5}}$ $|x|$ $|x|^{\frac{3}{4}}$ $x^{\frac{7}{9}}$ None of the above

18) If $f(x) = ax^2 + bx + 1$, and $f(1) = 0$, $f'(1) = 1$, then $b =$

2 -2 3 -3 None of the above

19) If a is a positive number, then the function $f(x) = ax^3 - 2x^2 + \frac{1}{a}x - 5$ is always decreasing on the interval

$[-\frac{1}{a}, \frac{1}{a}]$ $[\frac{1}{3a}, \frac{1}{a}]$ $[\frac{1}{2a}, \frac{1}{a}]$ $[\frac{1}{3a}, \frac{4}{5a}]$ None of the above

20) If a is a non-zero number, then the total number of inflection points for the function $f(x) = ax^4 + x^3 + \frac{1}{a}x^2 - 3x + 7$ is always

0 1 2 3 None of the above

21) You need to fence a rectangular play zone for children. What is the maximum area for this play zone if it is to fit into a right-triangular plot, with the perpendicular sides measuring 4m and 12m, respectively?

$9m^2$ $10m^2$ $11m^2$ $12m^2$ None of the above

22) If $f(x) = \sqrt[4]{\frac{x}{1-3x}}$, then $f'(x) = \frac{1}{4x^{\frac{3}{4}}(1-3x)^k}$, where $k =$

- $\frac{1}{2}$ $\frac{1}{4}$ $\frac{5}{4}$ $\frac{3}{2}$ None of the above

23) If $f(x) = (1+x)^{10}(1+2x)^{17}$, then $f'(0) =$

- 44 340 19 37 None of the above

24) The slope of the tangent line to the circle $x^2 + y^2 = 5x + 4y$ at the point (5,4) is

- $\frac{2}{3}$ $\frac{3}{4}$ $-\frac{5}{4}$ $-\frac{2}{3}$ None of the above

25) A small balloon is released at a point 150 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast (in feet per second) is the distance from the observer to the balloon increasing when the balloon is 50 feet high?

- 1.94 2.23 2.53 2.85 None of the above

