

Name \_\_\_\_\_

Math 116

**Long Calculus II**

**Final Exam**

**May 4, 2007**

Score \_\_\_\_\_

**Please do not separate the pages**

**Each problem carries a weight of 4 points**

1) How many (positive) solutions does the equation  $x^x = \frac{1}{x^2}$  have?

0    1    2    infinite    None of the above

2) For how many (positive) numbers  $a$  does  $\lim_{x \rightarrow 0} \frac{a^{3x} - 1}{x} = 1$  hold true

0    1    2    3    None of the above

3) The total number of critical points on the graph of the function  $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} - x$  is

0    1    2    4    None of the above

4) A freshly inoculated bacterial culture contains 100 cells. When the culture is checked 60 minutes later, it is determined that there are 450 cells present. Assuming exponential growth, how many minutes does it take for the number of cells to double?

25.73    26.45    27.65    28.82    None of the above

5) If  $\lim_{x \rightarrow 0} \frac{e^x + ax + b}{x^2}$  exists, then

$a = 2b$      $b = 2a$      $a = -b$      $a = b$     None of the above

6)  $\lim_{x \rightarrow 1^+} x^{\frac{1}{x-1}} =$

$\frac{2e^2}{5}$      $e$      $\frac{6}{e}$      $\frac{24}{e^2}$     None of the above

7) The global maximum of the function  $f(x) = \frac{\ln 2x}{x}$  is

$\frac{2}{e}$      $\frac{4}{5}$      $\frac{3}{1+e}$      $\frac{3}{4}$     None of the above

8)  $\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{\ln x} =$

- 0     $\frac{1}{2}$      $\frac{1}{e}$      $\infty$     None of the above

9) If  $f(x) = (x^2 - 3)^x$  then  $f'(2) =$

- 4     $\frac{1}{e}$      $e$     8    None of the above

10)  $\cos^2\left(\frac{\theta}{2}\right) =$

- $\cos \theta$      $1 + \cos \theta$      $\frac{1}{2}(1 + \cos \theta)$      $1 + \frac{1}{2} \cos \theta$     None of the above

11) For how many numbers  $a$  does  $\lim_{x \rightarrow a} \frac{1 - \cos(x - a)}{x^2 - 2ax + a^2}$  not exist?

- 0    1    2    infinite    None of the above

12) If  $f(x) = x \sin(x^3)$ , then  $f'\left(\sqrt[3]{\frac{\pi}{3}}\right) =$

- $\frac{\pi}{2} + \sqrt{3}$      $\pi + 2\sqrt{3}$      $\pi - \frac{\sqrt{3}}{2}$      $\frac{1}{2}(\pi + \sqrt{3})$     None of the above

13) The function  $f(x) = 2 \cos(3x + 5)$  is periodic and its period is

- $3\pi$      $\frac{2\pi}{3}$      $\frac{3\pi}{2}$      $\frac{3\pi}{5}$     None of the above

14) For how many numbers  $x$  does the identity  $\arcsin(\sin(x)) = \frac{\pi}{4}$  hold true?

- 1    2    4    8    None of the above

15) For how many numbers  $x$  does the identity  $\cos(\arcsin(x)) = \frac{1}{\sqrt{2}}$  hold true?

1    2    4    8    None of the above

16) If  $f(x) = \ln(\arctan(e^x))$ , then  $f'(0) =$

$\frac{\pi}{4}$      $\frac{\pi}{2}$      $\frac{2}{\pi}$      $\frac{4}{\pi}$     None of the above

17)  $\sum_{k=1}^{10} \left( \frac{1}{k} - \frac{1}{k+1} \right) =$

$\frac{19}{20}$      $\frac{16}{17}$      $\frac{10}{11}$      $\frac{9}{10}$     None of the above

18)  $\int_0^{\frac{3\pi}{2}} |\sin(x)| dx =$

$\pi$      $\frac{\pi}{2}$      $2\sqrt{2}$     3    None of the above

19) If  $F(x) = \int_0^{x^2} f(t) dt$ , and  $F'(x) = 2x^7 \sin(x^2)$ , then  $f(x) =$

$\frac{x^6}{2} \sin(x)$      $x^6 \sin(x^2)$      $x^3 \sin(x)$      $2x^3 \sin(x^2)$     None of the above

20)  $\int_1^2 x\sqrt{2-x} dx =$

$\frac{14}{15}$      $\frac{13}{12}$      $\frac{16}{17}$      $\frac{9}{10}$     None of the above

21)  $\int_1^2 x^3 \ln x dx =$

$8 \ln 2$      $(2 \ln 2) - \frac{3}{5}$      $(4 \ln 2) - \frac{15}{16}$      $(\ln 2) + \frac{23}{17}$     None of the above

22)  $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx =$

- $\frac{3\pi}{4}$      $\frac{3\pi}{16}$      $\frac{2\pi}{15}$      $\frac{2\pi}{9}$     None of the above

23) If  $F'(x) = \frac{1}{x^2 \sqrt{4-x^2}}$  for  $0 < x < 2$ , and  $F(2) = 0$ , then  $F(1) =$

- $\frac{2}{\sqrt{3}}$      $\frac{\sqrt{3}}{4}$      $-\frac{2}{\sqrt{3}}$      $-\frac{\sqrt{3}}{4}$     None of the above

24) The length of the curve  $y = \frac{1}{3}x^{\frac{2}{3}}$  from  $(0,0)$  to  $(9,9)$  is (approximately)

- 12.96    9.83    14.21    10.73    None of the above

25) The volume of the solid obtained by revolving the curve  $y = \sqrt{x}$  on the interval  $[0,4]$  about the  $x$ -axis is

- $2\pi$      $4\pi$      $8\pi$      $16\pi$     None of the above