Instructor's Name:____________________ **Your Name:**____________________
Section #:______________

122 FINAL EXAMINATION
Friday, May 7, 2007, 8:00 A.M.-12:00 NOON

INSTRUCTIONS: Do ALL 27 problems. For each question, choose only one best answer. **The only exceptions are problems 22 and 26, where you must mark the answers as instructed in the problem.** You may use a calculator, however **TI-89s, TI-92s, Voyage 200s or any other calculators capable of symbolic manipulation are not allowed.**

Inside the exam CIRCLE COMPLETELY and CLEARLY ONE BEST ANSWER to each problem, with the exception of problems 22 and 26.

SHOW and TURN IN ALL WORK inside this exam leading to your answers. Print your NAME on each page of this exam. Do not separate any of the pages of the exam.

1. (5 pts) Find the area of the region enclosed by $y = x^2$ and $y = 2 - x$.

   (a) $\frac{7}{6}$ (b) $\frac{9}{2}$ (c) $\frac{11}{6}$ (d) $\frac{21}{2}$

2. (5 pts) What is the volume of the solid obtained by rotating the region bounded by $y^2 = x$ and $x = 2y$ about the y-axis?

   (a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{16\pi}{15}$ (d) $\frac{64\pi}{15}$

3. (5 pts) A tank in the shape of the bottom half of a sphere of radius 1 m is filled with water. How much work does it take to pump all of the water out of the top of the tank? [Use the fact that gravity is $g \approx 9.8 \frac{m}{s^2}$ and the density of water is $\rho = 1000 \frac{kg}{m^3}$].

   (a) $\frac{343\pi}{20}$ J (b) $\frac{7\pi}{4}$ J (c) $17,150\pi$ J (d) $2,450\pi$ J

4. (5 pts) Suppose the velocity of a car is given in meters per second by the function $v(t) = 60\cos 3t + 40\sin 2t$. What is the average velocity of the car between $t = 0$ and $t = \frac{\pi}{2}$?

(a) $0\ \frac{m}{s}$ (b) $60\ \frac{m}{s}$ (c) $\frac{40}{\pi}\ \frac{m}{s}$ (d) $\frac{60}{\pi}\ \frac{m}{s}$

5. (5 pts) Evaluate the integral $\int_1^e x^2 \ln x\, dx$?

(a) $\frac{4e+12}{5}$ (b) $\frac{2e^3+1}{9}$ (c) e^2 (d) $\frac{e^3}{3} + \frac{e^2}{2}$

6. (5 pts) What is $\int \frac{1}{2+x^2} dx$?

(a) $\frac{x}{2} - \frac{1}{x} + c$ (b) $\ln|2+x^2| + c$ (c) $\frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$ (d) $\tan^{-1} \frac{x}{\sqrt{2}} + c$

7. (5 pts) What is $\int \sin^2\theta \cos^3\theta\, d\theta$?

(a) $\frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5} + c$ (b) $\frac{\cos^4\theta}{4} + \frac{\cos^6\theta}{6} + c$

(c) $\frac{\sin^3\theta\cos^4\theta}{12} + c$ (d) $\frac{\sin^3\theta\cos^3\theta}{3} + \frac{\sin^2\theta\cos^4\theta}{4} + c$

8. (5 pts) What is $\int \frac{1}{x^2 - 5x + 6} dx$?

(a) $\ln|x^2 - 5x + 6| + \mathrm{c}$ (b) $\ln\left|\frac{x-2}{x-3}\right| + c$ (c) $\ln\left|\frac{x-3}{x-2}\right| + c$ (d) $-\frac{1}{x} - \frac{\ln|x|}{5} + \frac{x}{6} + c$

9. (5 pts) Which one of the following statements is true?

   (a) If $f(x)$ is a decreasing and continuous function from $x = 0$ to $x = 1$, the left endpoint approximation will be an underestimate of $\int_0^1 f(x)\,dx$.

   (b) If $f(x)$ is a decreasing and continuous function from $x = 0$ to $x = 1$, the right endpoint approximation will be an underestimate of $\int_0^1 f(x)\,dx$.

   (c) The Midpoint Rule always gives a better estimate than the Trapezoidal Rule.

   (d) When using Simpson's Rule, the Midpoint Rule, or the Trapezoidal Rule increasing n can result in greater error.

10. (5 pts) Which one of the following improper integrals converges?

   (a) $\int_2^\infty \frac{1}{x\ln x}dx$ (b) $\int_1^\infty \frac{1}{x}dx$ (c) $\int_0^1 \frac{1}{x^2}dx$ (d) $\int_0^1 \frac{1}{\sqrt{x}}dx$

11. (5 pts) Let $y = \frac{2}{3}(x-1)^{\frac{3}{2}}$. What is the length of the curve from $x = 1$ to $x = 4$?

   (a) $\frac{16}{3}$ (b) $\frac{14}{3}$ (c) $\frac{21}{2}$ (d) 12

12. (5 pts) Consider the curve $y = 2\sqrt{x}$ from $x = 3$ to $x = 8$. What is the area of the surface obtained by revolving this around the x-axis?

   (a) $\frac{152\pi}{3}$ (b) $\frac{40\pi}{3}$ (c) $\frac{8\pi}{3}\left[8\sqrt{8} - 3\sqrt{3}\right]$ (d) $\frac{168\pi}{3}$

13. (5 pts) A cube with 0.1 m long sides is lying on the bottom of a pool in which the water is 1 m deep. What is the hydrostatic force on one of the sides of the cube (not the top or bottom)? [$g \approx 9.8\ \frac{m}{s^2}$ and $\rho = 1000\ \frac{kg}{m^3}$]

   (a) 93.1 N (b) 88.2 N (c) 9.5 N (d) 4.9 N

14. (5 pts) Let $x(t)$ be the solution of the initial value problem

$$\begin{cases} \dfrac{dx}{dt} = e^x e^t \\ x(0) = 0 \end{cases}$$

Then $x(1) =$

(a) 1 (b) $-\ln(2-e)$ (c) $e-1$ (d) $\ln(2-e)$

15. (5 pts) A tank initially contains 50 gallons of pure water. Through an inlet pipe brine solution, which contains 0.05 pounds of salt per gallon, starts to flow into the tank at a rate of 2 gallons per minute. Simultaneously, a drain is opened at the bottom of the tank so that well-mixed brine solution flows out at a rate of 2 gallons per minute. Let $m(t)$ be the amount of salt (in pounds) in the tank after t minutes. Then $m(t)$ satisfies the following differential equation:

(a) $\dfrac{dm}{dt} = 0.1 - \dfrac{m}{25}$ (b) $\dfrac{dm}{dt} = 2 - 2m$ (c) $\dfrac{dm}{dt} = 0$ (d) $\dfrac{dm}{dt} = 0.05 - m$

16. (5 pts) Estimate the solution of the given initial value problem at $t = 1$ using Euler's method with step size $h = 0.5$

$$\begin{cases} \dfrac{dx}{dt} = e^t + x \\ x(0) = 1 \end{cases}$$

(a) $x(1) \approx \dfrac{3}{2}$ (b) $x(1) \approx 2$ (c) $x(1) \approx 2 + \dfrac{\sqrt{e}}{2}$ (d) $x(1) \approx 3 + \dfrac{\sqrt{e}}{2}$

17. (5 pts) Let $y(t)$ be the solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} + 2ty = e^{-t^2} \\ y(0) = 1 \end{cases}$$

Then $y(1) =$

(a) $\dfrac{2}{e}$ (b) 2 (c) 0 (d) $\dfrac{1}{e}$

18. (5 pts) The parametric equations of a curve are given by

$$\begin{cases} x = \ln t - \dfrac{t^2}{2} \\ y = 2t \end{cases}$$

Where $1 \leq t \leq e$. Then the arc length of the curve is equal to

(a) $2e - \dfrac{1+e^2}{2}$ (b) $e - 1$ (c) $\dfrac{3-e^2}{2}$ (d) $\dfrac{1+e^2}{2}$

19. (5 pts) Consider the polar curves $r = 1$ and $r = 1 - \sin\theta$, where $0 \leq \theta \leq 2\pi$. Then the Cartesian coordinates of the intersections of these two curves are

(a) $(x, y) = (-1, 0), (1, 0)$
(b) $(x, y) = (0, -1), (0, 1)$
(c) $(x, y) = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$
(d) $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

20. (5 pts) The area of the region enclosed by the polar curve $r = 2\sin\theta$, $0 \leq \theta \leq 2\pi$ is

(a) $\dfrac{\pi}{2}$ (b) π (c) $\dfrac{3\pi}{2}$ (d) 2π

21. (5 pts) Which one of the following statements is true?

(a) The value of the series $1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots$ is 0 because the terms cancel in pairs.

(b) The alternating harmonic series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converges absolutely.

(c) It is possible that the interval of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$ is $(-2, 1)$

(d) If the partial sums of a series $\sum_{n=1}^{\infty} a_n$ are given by $s_n = \frac{n^2}{n^2 + n + 122}$, then $\sum_{n=1}^{\infty} a_n$ converges and its value is 1.

22. (2 pts each) **FOR EACH** of the following, answer "C" if the series converges or "D" if the series diverges.

(a) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{n}{2n^2 + 1}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(d) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(e) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(f) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

23. (5 pts) We want to use a partial sum s_n to estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ accurate to within 0.01 of the actual sum. How would you choose n that is the best?

(a) 10 (b) 8 (c) 7 (d) 4

24. (5 pts) The radius of convergence of $\sum_{n=0}^{\infty} \frac{x^n 2^n}{n+1}$ is

(a) 1 (b) 0 (c) ∞ (d) $\frac{1}{2}$

25. (5 pts) Let $f(x)$ be a function such that $f(0) = 1$ and $f^{(n)}(0) = (-1)^n$ for each positive integer n. Then the Taylor series of $f(x)$ centered at $x = 0$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$

26. (2 pts each) Match **EACH** function with one of these power series by writing the letter of the corresponding power series next to the function it represents.

(A) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + ...$ (B) $1 + 2x + 4x^2 + 8x^3 + ...$

(C) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...$ (D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$

(a) $\dfrac{1}{1-2x}$ ____

(b) $\ln(1+x)$ ____

(c) e^{-x} ____

(d) $x \sin x$ ____

27. (5 pts) Expressing $\displaystyle\int_0^1 \frac{e^x - 1}{x} dx$ as a series, we have

(a) $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...$

(b) $\frac{1}{2} \cdot \frac{1}{2!} + \frac{1}{3} \cdot \frac{1}{3!} + \frac{1}{4} \cdot \frac{1}{4!} + ...$

(c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$

(d) $1 + \frac{1}{2} \cdot \frac{1}{2!} + \frac{1}{3} \cdot \frac{1}{3!} + \frac{1}{4} \cdot \frac{1}{4!} + ...$