Instructor's Name:______________________ **Your Name:**______________________

MATH 221 FINAL EXAMINATION
Friday, May 4, 2007, 8:00 A.M.-12:00 NOON

INSTRUCTIONS: Do ALL problems, clearly circling your answer IN THE EXAM AND ON THE FRONT PAGE OF THE EXAM. Incorrect answers are not penalized. Calculators are permitted. SHOW AND TURN IN ALL WORK leading to your answers. Print your name on each page of the exam and on any scratch paper you use. **DO NOT SEPARATE** any pages of the exam.

1.	a.	b.	c.	d.	e.	16.	a.	b.	c.	d.	e.
2.	a.	b.	c.	d.	e.	17.	a.	b.	c.	d.	e.
3.	a.	b.	c.	d.	e.	18.	a.	b.	c.	d.	e.
4.	a.	b.	c.	d.	e.	19.	a.	b.	c.	d.	e.
5.	a.	b.	c.	d.	e.	20.	a.	b.	c.	d.	e.
6.	a.	b.	c.	d.	e.	21.	a.	b.	c.	d.	e.
7.	a.	b.	c.	d.	e.	22.	a.	b.	c.	d.	e.
8.	a.	b.	c.	d.	e.	23.	a.	b.	c.	d.	e.
9.	a.	b.	c.	d.	e.	24.	a.	b.	c.	d.	e.
10.	a.	b.	c.	d.	e.	25.	a.	b.	c.	d.	e.
11.	a.	b.	c.	d.	e.	26.	a.	b.	c.	d.	e.
12.	a.	b.	c.	d.	e.	27.	a.	b.	c.	d.	e.
13.	a.	b.	c.	d.	e.	28.	a.	b.	c.	d.	e.
14.	a.	b.	c.	d.	e.	29.	a.	b.	c.	d.	e.
15.	a.	b.	c.	d.	e.	30.	a.	b.	c.	d.	e.

1. What is the shortest distance from the point $P(0, 1, -5)$ to the line through the two points $P_1(1, 2, -4)$ and $P_2(4, 4, -6)$?

   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{42}}{\sqrt{17}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1 (e) None of the above

2. Find the distance from the point $P(-1, 1, -3)$ to the plane $-2x + 3y + z = 1$.

(a) $\frac{5}{2}$ (b) $\frac{\sqrt{13}}{\sqrt{2}}$ (c) $\frac{5\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{14}}$ (e) None of the above

3. Find the area of the triangle with vertices $A(2,0,-3)$, $B(3,1,0)$, and $C(5,2,2)$.

(a) $\frac{1}{2}\sqrt{3}$ (b) 2 (c) $\frac{3}{2}\sqrt{2}$ (d) $2\sqrt{2}$ (e) None of the above

4. Find the unit tangent vector $\vec{T}(t)$ for the curve

$$\vec{r}(t) = \langle 3\sin t, 6t, 3\cos t \rangle$$

at $t = \pi/2$.

(a) $\langle 0, 6/\sqrt{44}, -3/\sqrt{47} \rangle$ (b) $\langle 0, 6/\sqrt{45}, 3/\sqrt{45} \rangle$ (c) $\langle 0, 6/\sqrt{45}, -3/\sqrt{45} \rangle$
(d) $\langle 0, 6/\sqrt{42}, -3/\sqrt{42} \rangle$ (e) None of the above

5. Find the length of the curve given parametrically by $\vec{r}(t) = \langle e^t, e^{-t}, \sqrt{2}t \rangle$, $0 \leq t \leq 4$.

(a) $e^4 + e^{-4}$ (b) $e^4 - e^{-4}$ (c) $e^4 + e^{-4} + 4\sqrt{2}$ (d) $\sqrt{e^4 + e^{-4} + 2}$ (e) None of the above

6. The quantity

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$$

(a) exists and equals -1. (b) exists and equals 0. (c) exists and equals 1.
(d) does not exist. (e) None of the above

7. If $z = x^2 - xy + 7y^2$ and (x, y) changes from $(2, 1)$ to $(1.95, 1.07)$, find the differential dz.

(a) 0.96 (b) 0.69 (c) −0.69 (d) −0.96 (e) None of the above

8. Use the Second Derivative Test to classify the point $(1, 1/2)$ for the function $f(x, y) = x^3 - 6xy + 8y^3$.

(a) Local minimum(b) Local maximum(c) Saddle point(d) The Second Derivative Test is inconclusive
(e) The point $(1, 1/2)$ is not a critical point

9. Find the absolute (global) maximum value of the function $f(x, y) = x^2 + xy + y^2$ on the disk D given by $x^2 + y^2 \leq 9$.

(a) 9/2 (b) 9 (c) 27/2 (d) 27 (e) None of the above

10. Which of the following integrals gives the area of the circle $x^2 + y^2 = x$?

(a) $\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} r\, dr\, d\theta$
(b) $\int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=\cos\theta} r\, dr\, d\theta$
(c) $\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=\sin\theta} r\, dr\, d\theta$
(d) $\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=\cos\theta} r\, dr\, d\theta$
(e) None of the above

11. Express

$$\int_{x=0}^{x=1/2} \int_{y=\sqrt{3}x}^{y=\sqrt{1-x^2}} x \, dy \, dx$$

as an iterated integral in polar coordinates.

(a) $\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=1} r^2 \cos\theta \, dr \, d\theta$
(b) $\int_{\theta=0}^{\theta=\pi/3} \int_{r=0}^{r=1} r \cos\theta \, dr \, d\theta$
(c) $\int_{\theta=\pi/3}^{\theta=\pi/2} \int_{r=0}^{r=1} r \cos\theta \, dr \, d\theta$
(d) $\int_{\theta=\pi/6}^{\theta=\pi/2} \int_{r=0}^{r=1} r^2 \cos\theta \, dr \, d\theta$
(e) None of the above

12. Evaluate by reversing the order of integration:

$$\int_{y=0}^{y=1} \int_{x=4y}^{x=4} e^{x^2}\, dx\, dy.$$

(a) $\pi^{16} - 1$ (b) $e^{16} - 1$ (c) $\frac{\pi^{16}-1}{8}$ (d) $\frac{e^{16}-1}{8}$ (e) None of the above

13. Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9, z = 0$.

(a) 40.5π (b) -7.5π (c) 68.5π (d) -68.5π (e) None of the above

14. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 64$ that lies above the plane $z = 3$.

(a) $80 - \pi$ (b) 80π (c) $80\sqrt{2}\pi$ (d) $\pi/80$ (e) None of the above

15. Evaluate $\int_C xy^4\,ds$, where C is the right half of the circle $x^2 + y^2 = 4$, oriented counterclockwise.

(a) 0.8 (b) 267.0 (c) 25.6 (d) 97.4 (e) None of the above

16. Let $\vec{F} = \vec{\nabla} f$ and $f(x,y) = \sin(x - 8y)$. Let C be the line segment from $(0,0)$ to $(\pi/2, \pi/2)$. Choose the correct statement.

(a) $\int_C \vec{F} \cdot d\vec{r} = 0$.
(b) $\int_C \vec{F} \cdot d\vec{r} = 1$.
(c) $\int_C \vec{F} \cdot d\vec{r}$ depends on the parameterization of C.
(d) $\int_C \vec{F} \cdot d\vec{r} = \pi$.
(e) None of the above

17. Find the value of the line integral $\int_\gamma (1+y)dx + (x+y)dy$ where $\gamma(t) = (t^2 e^{t^2}, t^4 - t)$, $0 \le t \le 1$.

(a) $\frac{e^2}{2}$ (b) e (c) $\frac{e-1}{2}$ (d) e^2 (e) None of the above

18. Suppose the force at the point (x, y) is $\vec{F}(x, y) = \langle 2x, -y \rangle$. Let γ be the curve consisting of the line segment from $(0, 1)$ to $(1, 0)$. Find the work done by the force $\vec{F}$ along the curve γ. [The magnitude of the force is in lbs. and distance is in ft.]

(a) $1 ft \cdot lb.$ (b) $2 ft \cdot lb.$ (c) $\frac{3}{2} ft \cdot lb.$ (d) $\frac{5}{2} ft \cdot lb$ (e) None of the above

19. Find the value of the line integral of the vector field

$$\vec{F}(x,y) = (3x^2y + e^x\cos(x+1))\vec{\imath} + (2x + x^3 + \log(y+3))\vec{\jmath}$$

along the closed curve $\vec{\gamma}(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath}, \quad 0 \le t \le 2\pi$.

(a) 0 (b) π (c) 2π (d) -2π (e) None of the above

20. Evaluate

$$\oint_C (\ln x + y)\, dx - x^2\, dy,$$

where C is the rectangle with vertices $(1,1)$, $(3,1)$, $(3,4)$, and $(1,4)$, oriented counterclockwise.

(a) 0 (b) 6 (c) -12 (d) 30 (e) None of the above

21. Find the value of the line integral of the vector field

$$\vec{F}(x,y) = (2 + xy^2)\vec{i} + (x^2y + x)\vec{j}$$

along the clockwise curve $y = \sqrt{4 - x^2}$, $-2 \leq x \leq 2$.

(a) 0 (b) $8 - 2\pi$ (c) 6π (d) $2 - 2\pi$ (e) None of the above

22. Let Σ be the surface $x = v^2, \quad y = u^2, \quad z = v + 3u^2$. Find the tangent plane at (1,1,4).

(a) $x + 6y - 2z = -1$ (b) $2x - 4y + 3z + 2 = 3$ (c) $-3x - y + 2z - 4 = 0$
(d) $x - 3y + 2z + 4 = 0$ (e) None of the above

23. Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F}(x, y, z) = \langle 2xy + 3x^2 z, x^2 + 2y^4, y^3 + z^2 \rangle$$

and C is the intersection of the plane $x + y + 2z = 1$ and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise when viewed from above.

(a) 0 (b) 4π (c) -8π (d) 12π (e) None of the above

24. Let S be the oriented surface consisting of the part of the hemisphere $z = \sqrt{1 - x^2 - y^2}$ above the xy-plane with upward normals. Let $\vec{F}$ be the vector field

$$\vec{F}(x, y, z) = \langle x^2, y^2, z^2 \tan(xy) \rangle.$$

Use Stokes' Theorem to find the value of the surface integral $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$.

(a) 4π (b) 2π (c) 6π (d) 0 (e) None of the above

25. Let S be the oriented surface consisting of the part of the parabaloid $z = 4 - x^2 - y^2$ above the xy-plane with outward normals. Let $\vec{F}$ be the vector field

$$\vec{F}(x, y, z) = \langle -y + z(2 + e^x), x + y^3 z + z^2, z + xz^3 \rangle.$$

Find the value of the surface integral $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$.

(a) 4π (b) 2π (c) 6π (d) 8π (e) None of the above

26. Let S be the sphere given by $x^2 + y^2 + z^2 = 1$ with outward normals. Let $\vec{F}$ be the vector field

$$\vec{F}(x,y,z) = \langle x + y^4 + e^{yz}, y + z^2 + \log(1+x^2), y + z\rangle.$$

Use the Divergence Theorem to find the value of the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

(a) 16π (b) $\frac{32\pi}{3}$ (c) $\frac{18\pi}{3}$ (d) 4π (e) None of the above

27. Let S be the sphere $x^2 + y^2 + z^2 = a^2$ where $0 < a$ with the outward normals. Let $\vec{F}$ be the vector field

$$\vec{F}(x, y, z) = (\sqrt{x^2 + y^2 + z^2})\langle x, y, z\rangle.$$

Find the value of the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

(a) $4\pi a^4$ (b) $2\pi a^3$ (c) 6π (d) $8\pi a^2$ (e) None of the above

28. Let S be the ellipsoid $x^2/4 + y^2 + z^2/9 = 1$ with the outward normals. Let $\vec{F}$ be the vector field

$$\vec{F}(x, y, z) = \langle x, y, z \rangle / (x^2 + y^2 + z^2)^{3/2}.$$

Find the value of the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

(a) 8π (b) 2π (c) 0 (d) 4π (e) None of the above

29. Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = y\vec{i} - x\vec{j} + \vec{k}$ and let S be the surface of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 0$ with upward normals. Evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dS$.

(a) 0 (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\pi\sqrt{2}$ (e) None of the above

30. Let S be the part of the paraboloid given by $z = 12 - x^2 - y^2$ above the plane $z = 11$ with upward normals. Let C be the oriented boundary of S in which the above paraboloid intersects the plane $z = 11$. Find the value of the line integral of the vector field $\vec{F}$ given by

$$\vec{F}(x, y, z) = \langle -y/(x^2 + y^2), x/(x^2 + y^2), \cos(z^2) + z^3 + 4 \rangle$$

along the closed curve C.

(a) 16π (b) $\frac{32\pi}{3}$ (c) 0 (d) 2π (e) None of the above