ANSWER SHEET **MATH 224 FINAL EXAM**

Name:________________________________ Instructor: ______________________

Friday May 4, 2007 - 8:00am - Noon

This exam contains **27** problems. The first **18** are multiple choice and are **6** points each. Problems **19 to 22** are true or false and are **3** points each.

Record the answers to **1 to 22** on the cover sheet. Clearly mark one answer only. If two or more answer are marked, no credit will be given. No partial credit will be given if a wrong answer is marked.

Problems **23 to 27** are to be worked in the space provided and are **16** points each. Show all of your work clearly for these problems because partial credit will be given. The last sheet is a Laplace transform table.

Calculators are allowed. Good luck!

1. Do NOT SEPARATE answer sheet from rest of test.
2. Work and CIRCLE the answer to each problem INSIDE this test.
3. Circle your answer a SECOND TIME on this page.

QUESTION	ANSWER					QUESTION	ANSWER				
1.	a.	b.	c.	d.	e.	12.	a.	b.	c.	d.	e.
2.	a.	b.	c.	d.	e.	13.	a.	b.	c.	d.	e.
3.	a.	b.	c.	d.	e.	14.	a.	b.	c.	d.	e.
4.	a.	b.	c.	d.	e.	15.	a.	b.	c.	d.	e.
5.	a.	b.	c.	d.	e.	16.	a.	b.	c.	d.	e.
6.	a.	b.	c.	d.	e.	17.	a.	b.	c.	d.	e.
7.	a.	b.	c.	d.	e.	18.	a.	b.	c.	d.	e.
8.	a.	b.	c.	d.	e.	19.	True		False		
9.	a.	b.	c.	d.	e.	20.	True		False		
10.	a.	b.	c.	d.	e.	21.	True		False		
11.	a.	b.	c.	d.	e.	22.	True		False		

Problems 23-27: work in the space provided in the exam

Math 224 FINAL EXAM — Spring 2007

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1. Let $x(t)$ be the solution of the initial value problem

$$\begin{cases} x' = -3x(2-x)(x+2)(x-1), \\ x(0) = 1.5. \end{cases}$$

Then the limit $\lim_{t\to\infty} x(t)$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 3 (e) none of the above

2. Consider the planar (2×2) system

$$\begin{cases} x' = x - 2y, \\ y' = 2x + y. \end{cases}$$

Then the origin $(0, 0)$ in the phase plane is:

(a) a saddle point (b) a nodal sink (c) a nodal source

(d) a spiral sink (e) none of the above

3. Suppose the linear planar (2×2) system $\mathbf{y}' = A\mathbf{y}$ has two solutions

$$\mathbf{y}_1(t) = e^{-3t}\begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{y}_2(t) = e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Which of the following is not a solution of $\mathbf{y}' = A\mathbf{y}$?

(a) $e^{-3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (b) $e^{-3t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (c) $e^{-3t}\begin{pmatrix} 2 \\ 4 \end{pmatrix} - e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(d) $4e^{-3t}\begin{pmatrix} 2 \\ 4 \end{pmatrix} + 7e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (e) none of the above

8. The inverse Laplace transform of $\dfrac{s^2}{s^4-1}$ is:

(a) $\frac{1}{2}(\cosh t + \sin t)$ (b) $\frac{1}{2}\sinh t + \sin t$ (c) $\frac{1}{4}e^t + \frac{1}{2}(\sin t - \cos t)$

(d) $\frac{1}{2}(\sinh t + \sin t)$ (e) none of the above

Recall that $\cosh t := \dfrac{e^t + e^{-t}}{2}$ and $\sinh t := \dfrac{e^t - e^{-t}}{2}$.

9. Let $x(t)$ be the solution of the initial value problem $\begin{cases} x' = tx + \dfrac{t}{x}, \\ x(1) = 1. \end{cases}$

Then

(a) there is no interval containing both points $t = 1$ and $t = -1$ in which the solution is defined

(b) the solution is not unique so there is no way to determine the value of $x(-1)$

(c) $x(-1) = 1$ (d) $x(-1) = -1$ (e) none of the above

10. Let $y(t)$ be the solution of the initial value problem $\begin{cases} y' = y - \dfrac{y}{t} + \dfrac{1}{t}, \\ y(1) = 0. \end{cases}$

Then

(a) there is no interval containing both points $t = 1$ and $t = 2$ in which the solution is defined

(b) $y(2) = \ln(e^2 - e + 1)$ (c) $y(2) = \dfrac{e-1}{2}$ (d) $y(2) = 1$

(e) none of the above

11. Let $x(t)$ be the solution of the initial value problem $\begin{cases} 4x'' + 4x' + x = 0, \\ x(1) = 1, \quad x'(1) = 0. \end{cases}$

Then $x(0)$ is equal to:

(a) $-\dfrac{e^{\frac{1}{2}}}{2}$ (b) 0 (c) $\dfrac{e^{\frac{1}{2}}}{2}$ (d) $\dfrac{e^{-\frac{1}{2}}}{2}$ (e) none of the above

16. Let $\mathbf{x}(t)$ be the solution of the system $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x}$ with the initial condition $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Which of the following is true?

(a) $\mathbf{x}(t)$ spirals towards the origin as $t \to \infty$, moving clockwise

(b) $\mathbf{x}(t)$ spirals towards the origin as $t \to \infty$, moving counterclockwise

(c) $\mathbf{x}(t)$ converges to the origin as $t \to \infty$, tangent to some line

(d) $\mathbf{x}(t)$, $t \geq 0$, is unbounded (e) none of the above

17. The general solution of the system $\begin{cases} x' = -2x + y \\ y' = -x - 4y \end{cases}$ is:

(a) $C_1 e^{-3t} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (b) $C_1 e^{-3t} \begin{pmatrix} 2 \\ -2 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(c) $C_1 e^{-3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ (d) $C_1 e^{-3t} \begin{pmatrix} -2 \\ 2 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(e) none of the above

18. Consider the Lorenz system

$$\begin{cases} x' = -x + y, \\ y' = rx - y - xz, \\ z' = -z + xy, \end{cases}$$

where r is a positive constant. The origin is a stable equilibrium point of this system provided:

(a) $r > 1$ (b) $1 < r < 4$ (c) $r \neq 1$ (d) $r < 1$ (e) none of the above

23. Use polar coordinates to show that the system

$$\begin{cases} x' = y + x\left(3 - \sqrt{x^2+y^2} - \dfrac{2}{\sqrt{x^2+y^2}}\right), \\ \\ y' = -x + y\left(3 - \sqrt{x^2+y^2} - \dfrac{2}{\sqrt{x^2+y^2}}\right), \end{cases}$$

has two limit cycles. Find them and check their stability.

26. Using the Laplace transform, solve the initial value problem

$$\begin{cases} y'' + 2y' + 2y = \cos(2t), \\ y(0) = 0, \quad y'(0) = 1. \end{cases}$$

Note that **no** (partial) **credit** will be given if the problem is solved using the method of undetermined coefficients or any other technique.

A small table of Laplace transforms

$$\begin{aligned}
\mathcal{L}\{1\}(s) &= \frac{1}{s} \\
\mathcal{L}\{t^n\}(s) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}\{\sin(at)\}(s) &= \frac{a}{s^2+a^2} \\
\mathcal{L}\{\cos(at)\}(s) &= \frac{s}{s^2+a^2} \\
\mathcal{L}\left\{e^{at}\right\}(s) &= \frac{1}{s-a} \\
\mathcal{L}\left\{e^{at}\sin(bt)\right\}(s) &= \frac{b}{(s-a)^2+b^2} \\
\mathcal{L}\left\{e^{at}\cos(bt)\right\}(s) &= \frac{s-a}{(s-a)^2+b^2} \\
\mathcal{L}\left\{t^n e^{at}\right\}(s) &= \frac{n!}{(s-a)^{n+1}} \\
\mathcal{L}\{H(t-c)f(t-c)\}(s) &= e^{-cs}F(s), \quad \text{where } F(s) = \mathcal{L}\{f(t)\}(s) \\
\mathcal{L}\{\delta(t-c)\}(s) &= e^{-cs}
\end{aligned}$$