MATH 301/601, Final Exam, May 5, 2007

1. You are dealt a poker hand. Below is an expression for the probability to get three of a kind but one factor X is missing. What is X?

$$P(\text{three of a kind}) = \frac{13 \cdot \binom{4}{3} \cdot X \cdot 4 \cdot 4}{\binom{52}{5}}$$

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2. You have estimated that there is a 5% chance that something is wrong with your car each time you start it. If something is wrong, there is a 75% chance that the "check engine" light comes on. If nothing is wrong, there is still a 10% chance that the light comes on in error.

(a) What is the probability that the light comes on when you start your car?

(b) If the light comes on, what is the probability that there is something wrong?

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3(a) Let $X \sim \text{bin}(n, p)$ have mean 30 and variance 21. What are n and p?

(b) Roll a pair of dice five times and let X be the number of times you get a double six. What is the distribution of X?

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4. It was recently reported that there is about a 50-50 chance that the Gulf Coast will be hit by a hurricane in the 2007 season June 1–November 30. Assume that hurricanes during the season hit the Gulf Coast according to a Poisson process with rate λ hurricanes/month.

(a) Find the value of λ (be careful with the time units).

(b) Let X be the number of hurricanes that hit the Gulf Coast during the months of August and September. What is the distribution of X (name and parameter(s))?

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5. Let $R \sim$ unif $[0, 1]$ and let V be the volume of a sphere with radius R. Find $E[R]$ and $\text{Var}[R]$.

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6. Let $X \sim N(0, 1)$ and let $Y = X^2$.

(a) Find $P(X > 1)$ and $P(-1 \leq X \leq 2)$.

(b) Find $P(Y = 1)$ and $P(Y \leq 1)$.

(c) Find $P(Y \leq 4 | X > 1)$.

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7. The pair (X, Y) has a uniform distribution on the triangle with corners in $(0,0), (1,0)$, and $(1,2)$. Find

(a) the joint pdf $f(x,y)$ (including range)

(b) the marginal pdf $f_X(x)$ (including range)

(c) the conditional pdf $f_Y(y|x)$ (including range)

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8. Let X have a uniform distribution on $[0,1]$ and given $X = x$, let Y have a uniform distribution on $[-x, x]$.

(a) Find the joint pdf of (X, Y) and indicate its range in a figure.

(b) Compute $P(Y \leq X/2)$.

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9. Let $X_1, ..., X_n$ be a sample from a distribution with pdf

$$f(x) = ax^{a-1}, \quad 0 \leq x \leq 1$$

where a is an unknown parameter. Find

(a) the method of moment estimator of a

(b) the maximum likelihood estimator of a.

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10. A lake contains equal numbers of two species of fish A and B. The A-fish have mean weight 10 and variance 4, and the B-fish have mean weight 8 and variance 2. A fish is caught randomly from the lake and its weight denoted by X. Find the variance of X.

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11. Write a short poem about the course. Any style is acceptable (limerick, haiku, iambic pentameter, free form, sonnet,...).