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Advanced Calculus

1. Let $f_1(x) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$f_2(x,y) = \begin{cases} \dfrac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Discuss the continuity of each of these functions at (0,0).

2. Consider the transformation of the plane given by

$$u = \frac{x^2-y^2}{\sqrt{x^2+y^2+1}}$$

$$v = \frac{2xy}{\sqrt{x^2+y^2+1}}$$

a. Show that the transformation cannot have a global inverse.

b. Use the inverse function theorem to show that it has a local inverse near all points $(x_0,y_0) \neq (0,0)$.

August 1987

3. Let $p(x,y,z)$ and $q(x,y,z)$ be two continuously differentiable function defined in R^3.

Let $\gamma(t) = \{ \frac{1}{2} \cos t, \frac{1}{2} \sin t, \frac{\sqrt{3}}{2}) \}$

Write $\int_\gamma pdx + qdy$ as an integral over the surface given by $x^2 + y^2 + z^2 = 1$, $z \geq \frac{\sqrt{3}}{2}$.