**Linear Algebra and Advanced Calculus Written Examination**

**Thursday, December 1, 1994 – 9:00 A.M. - 1:00 P.M.**

**Gibson Hall, Room 415**

1. Find the equation of the plane tangent to the surface $S$ defined by the equation

$$x^3 + y^3x + z^4y + z = 12$$

at the point $(2,1,1)$.

2. $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x,y) = (x^2 + e^{3y}, 4y + xy)$$

   a) Find the affine transformation [linear transformation plus a translation] which approximates $f$ best for $(x,y)$ near the point $(1,0)$.

   b) Prove that $f$ has a differentiable inverse in a neighborhood of the point $f(2,0) = (5,0)$

   c) Prove that $f$ does not have a differentiable inverse in a neighborhood of the point $f(0,0) = (1,0)$.

3. Let $F(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$ be a vector field defined on $\mathbb{R}^2\backslash\{(0,0)\}$ and consider the curve $\gamma(t) = (\cos(t), \sin(t)) \quad 0 \le t \le 2\pi$.

   a) Calculate the line integral of $F$ along $\gamma$, $\int_\gamma F$.

   b) Prove that there is no function $f$ defined on $\mathbb{R}^2\backslash\{(0,0)\}$ satisfying $\nabla f = F$.

   c) Let $\beta$ be a differentiable closed curve which does not intersect itself and with the property that $(0,0)$ is neither on the curve nor is contained in the region bounded by the curve. Prove that $\int_\beta F = 0$.

4. a) Let $D$ and $D'$ be bounded regions of $\mathbb{R}^2$ and let $D' \xrightarrow{f} D$ be a 1-to-1, onto, differentiable map. Let $g$ be a continuous function on $D$. State the change of variables formula expressing the double integral $\iint_D g$ in terms of a double integral over $D'$.

b) Use the change of variables formula to evaluate

$$\iint_D \frac{1}{1+x^2+y^2}$$

where $D$ is the right half of the disc with center $(0,0)$ and radius $1$.

5. Find the absolute max. points and absolute min. points of $f(x,y) = 2x^2 + y^2 - y$ on the set $\{(x,y) \mid x^2 + y^2 \leq 1\}$.

6. Find the absolute max. points and absolute min. points of $f(x,y,z) = xy - 2yz$ subject to the constraint $x^2 + y^2 + z^2 = 1$.