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File

TOPOLOGY EXAM

August ~~22~~ 21, 1991

No work $\Longrightarrow$ no credit.

A lot of work $\Longrightarrow$ credit (may be).

I. A. Define:

a) Hausdorff space

b) normal space

c) compact space

B. Give a complete (and self-contained) proof of the following result:

"A compact space is normal"

C. Can you give an example of a metric space which is not normal?

II. Show that a compact metric space X can not be isometric with any of its proper subspaces.

Hint. Start with a point $x_0 \in X$ which is not in the range of the isometry f. Look at $x_1 = f(x_0)$. If you are still in troubles iterate.

III. A. Define the notion of

a. connected space

b. locally connected space

c. path connected space.

B. Show that path connected space is connected.

C. Can you give examples of spaces which are:

a. connected but not path connected

b. path connected but not locally connected

c. locally connected but not connected

d. path connected but not locally connected.

IV. 1. Discuss the construction of $\pi_1(X, x_0)$.

2. Describe the homomorphism induced by the continuous map $f : (X, x_0) \to (Y, y_0)$.

3. Show (beyond any reasonable doubt) that you know why for a path connected space X the groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$, $x_0 \neq x_1$, are isomorphic.

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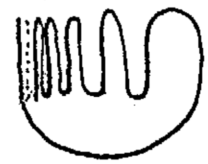
IV. 4. Did you hear about the Van-Kampen Theorem? If so, write it down explaining the terms which are involved in its formulation.

5. Please share with us your knowledge concerning the fundamental groups of

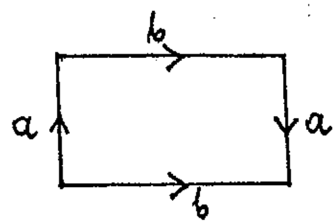
$\pi_1(R^2) =$

$\pi_1($  $) =$

$\pi_1(R^2 \times S^1 \times (R^2 - \{\text{pt}\})) =$

$\pi_1(K) =$

$K =$ Klein bottle :=



V. A. List all covering spaces of $\mathbf{R}P^2 \times \mathbf{R}P^2$; $\mathbf{R}P^2$ is the projective plane, i.e. $\mathbf{R}P^2 = S^2\,/\text{Antipodism}$.

B. Prove that any map from $\mathbf{R}P^2$ into the torus $T = S^1 \times S^1$ is nullhomotopic.

VI. Assume $(X; X_1, X_2)$ is exact, $X = X_1 \cup X_2$.

a. Write down the Mayer-Vieetoris exact sequence associated with $(X; X_1, X_2)$.

b. Think about the Klein bottle K as obtained by gluing two copies of the Möbius band along their boundaries. Use the M-V sequence to compute $H_*(K; \mathbf{Z})$. Please, do not let your teacher down.

c. How could you "predict" $H_1(K; \mathbf{Z})$ knowing $\pi_1(K)$?

VII. There is a hope after all; an easy problem.

A. Let $\mathbf{C}P^n$ be the complex projective space. Compute the Euler characteristic of the pair $(\mathbf{C}P^4, \mathbf{C}P^2)$. In order to do it, first:

a. Recall the notion of the Euler characteristic for a space X.

b. Recall the relation between $\chi(X; A)$, $\chi(A)$ and $\chi(X)$.

B. Is it true that R^4 and R^3 are homotopy equivalent but $R^4 - \{\text{pt}\}$ and $R^3 - \{\text{pt}\}$ are not?