August 92

Topology Exam

1. State and prove the ~~Basic~~ Baire Category Theorem.

2. Define *locally compact space.* Let X be a locally compact space that is not compact. Define and discuss the *one-point compactification* of Y.

3. Define *connected*, *path connected* and *locally path connected.* If X is a locally path connected space, prove that every connected open subset of X is path connected.

4. Let $\sim$ be an equivalence relation on a topological space X. Define the *quotient topology* for $X/\sim$.

   Let $\pi : X \to X/\sim$ be the projection map. Prove that a function f from $X/\sim$ to a space Y is continuous if and only if $f \circ \pi$ is continuous.

5. Define $\pi_1(X, x_0)$. What is its group operation? inverses? identity?

   If $f : (X, x_0) \to (Y, y_0)$ is a map, define the induced homomorphism of fundamental groups.

6. (a) Define $p : (E, e_0) \to (B, b_0)$ *is a covering map.* Prove: If E is simply connected, then there is a bijection

$$\varphi : \pi_1(B, b_0) \to p^{-1}(b_0) .$$

   (b) Define the projective plane P^2. Use a covering space argument to determine $\pi_1(P^2)$. Can you use this last answer to determine $H_1(P^2)$?

7. (a) Let $H_p(K)$ denote the p^{th} homology group (simplicial or singular – you choose) of K with integral coefficients. Define $H_p(K)$ by first defining p-*chain*, p-*cycle*, and p-*boundary* and then p^{th}-*homology group.*

(b) State (no proofs) the homology groups below, for all integers $q > 0$:

(i) $H_q(S^n)$

(ii) $H_q(E^n)$

(iii) $H_q(S^n, S^n)$

(iv) $H_q(E^n, S^{n-1})$

(v) $H_q(S^n, x)$

(vi) H_q(figure 8)

(vii) H_q(n distinct points)

(viii) H_q(torus)

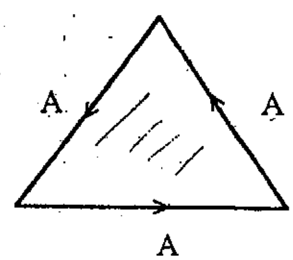
(c) What is the relationship between $H_q(x)$ and $\widetilde{H}_q(x)$?
between $H_q(X, A)$ and $\widetilde{H}_q(X, A)$, A nonempty?

(d) State the excision property.

8. Compute (i.e., show some work) the homology groups of

(a) the Klein bottle

(b) the quotient space obtained in the triangle (including interior) at right with identification on the boundary as indicated.



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