Written Qualifying Exam in Topology

January 1994

1. Let $X = \{a, b, c\}$ and let $\mathcal{T}$ be a topology on X given by
$$\mathcal{T} = \{X, \phi, \{a,b\}, \{a,c\}, \{a\}\}$$
Let $A = \{a, c\}$
Find (i) $\text{int}(A)$
(ii) $\overline{A}$
(iii) ∂A

2. (i) Show that every compact Hausdorff space is normal.
(ii) Is every normal space compact? (justify your answer).

3. Show that the space $[0, \infty)$, (with the ~~standard~~ standard topology) is connected.

4. Compute the fundamental group of the real projective plane $\mathbb{R}P^2$. (You can use any description of $\mathbb{R}P^2$).

5. What are the homology groups (with $\mathbb{Z}$ coefficients) of
(i) $X = R^2 - \{0,0\}$
(ii) $Y = R^2 - (\{0,0\} \cup \{0,1\})$
(iii) $Z = S^1 \times \mathbb{R}^3$.

6. Can you compute the homology groups of S^1, by using the definition and standard triangulation of S^1 as

4. Use the Mayer-Vietoris exact sequence to compute the homology groups ($\mathbb{Z}$-coefficients) of the torus $T^2 = S^1 \times S^1$.