

# Topology Final

Spring 2008

1. Let  $\mathbb{P}^n$  be the  $n$ -dimensional projective space. Show that  $\mathbb{P}^n$  is not a retract of  $\mathbb{P}^m$  for  $m > n$ .
2. Compute the homology of the “dunce cap.” The dunce cap is the space obtained from a triangle by identifying the three edges by  $(a, b) \sim (b, c) \sim (a, c)$ .
3. Compute the homology groups  $T^2 \sqcup P^2$ . Here  $\sqcup$  denotes the connected sum.
4. Determine whether there exists a short exact sequence

$$0 \longrightarrow \mathbb{Z}_4 \longrightarrow \mathbb{Z}_8 \oplus \mathbb{Z}_2 \longrightarrow \mathbb{Z}_4 \longrightarrow 0$$

5. Let  $SX$  be the suspension of a space  $X$ . Show by using the Mayer Vietoris sequence that there are isomorphism  $H_n(SX) \cong H_{n-1}(X)$  for all  $n$ .