

1. Let  $z = f(x, y)$ . Find the slope of the tangent line, a tangent vector, a normal vector, and the equation of the tangent line at a given point.
2. TANGENT PLANES.
  - a) Find the equation of the tangent plane (or the equation of the line perpendicular to  $f$ ) to  $f(x, y) = \dots$  at  $(a, b)$ . See 15.4.  $f_x(a, b)(x - a) + f_y(a, b)(y - b) - (z - f(a, b)) = 0$ .
  - b) Find the equation of the tangent plane to something like  $x^2 + 2y^2 - z^4 - z^2 = 4$  at the point  $(a, b, c)$ . See 15.6.  $F_x(a, b, c)(x - a) + F_y(a, b, c)(y - b) + F_z(a, b, c)(z - c) = 0$ .
3. DIFFERENTIAL. Let  $f(x, y) = \dots$ . Approximate  $f(1.1, -.02)$ . Linearize  $f(x, y)$  at the point  $(2, -1)$ .
4. IMPLICIT DIFFERENTIATION. page 956, 957.
5. CHAIN RULE. Find  $\frac{\partial f(x, y)}{\partial u}$  and  $\frac{\partial f(x, y)}{\partial v}$  where  $x$  and  $y$  are functions of  $u$  and  $v$ .
6. DIRECTIONAL DERIVATIVE. Let  $f(x, y)$  be a continuous function. Let  $(a, b)$  be a point in the domain of  $f$ . What is the rate of change of  $f$  in the direction of the vector  $u = \langle 2, -3 \rangle$ ? What is the greatest rate of change of  $f$  and in which direction is it happening?
7. LOCAL-GLOBAL MAX-MIN.
  - a) Find and classify the critical points of  $f(x, y)$ : set the partial derivatives  $= 0$ , then solve for  $x$  and  $y$ .
  - b) Find the global max-min of  $f$  on the closed bounded set  $D$ : if  $D$  is a square, or a triangle, or any polygon find out what  $f$  looks like on each edge and maximize (minimize) it on each edge. If  $D$  is the inside of some curve (circle, ellipse, etc..) use Lagrange multipliers or parametrize the curve.
8. LAGRANGE MULTIPLIERS. Find the max-min of a function  $f(x, y)$  subject to  $g(x, y) = k$ .
9. DOUBLE INTEGRALS. Integrate a double integral over a region  $D$ . You need to be able to recognize if  $D$  is of type I or II or both, or a polar region. If  $D$  is of both types, you need to be able to choose the correct order of integration so you can calculate the integral.

Good luck!