Double Integrals:

1. Know how to change the order of integration $dxdy \leftrightarrow dydx$.

2. Know how and when to change to polar coordinates. For instance, if you see something like $\sqrt{9-x^2}$, x^2+y^2 , $\sqrt{17-y^2}$, $\sqrt{x^2+y^2}$ inside the integral or in the limits of integration, then you should think of changing to polar coordinates.

3. $\int\int_R dA =$ Area of R.

4. If $f(x,y)$ is positive, then $\displaystyle\int\int_R f(x,y)dA =$ volume under f over the region R.

5. $\displaystyle\int\int_R f_2(x,y) - f_1(x,y)dA =$ volume between f_1 and f_2.

Triple Integrals:

1. $\displaystyle\int\int\int_E dV =$ volume of the solid E.

2. $\displaystyle\int\int\int_{f_1(x,y)}^{f_2(x,y)} dV =$ volume between f_1 and f_2. (Notice the similarity with 5 above).

3. Know how to change the order of integration $dzdydx \leftrightarrow dxdzdy \leftrightarrow dydxdz$...

4. Know how and when to change to cylindrical-spherical coordinates. For instance, if you have something like $\displaystyle\int_0^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \int_{-\sqrt{25-x^2-y^2}}^{\sqrt{25-x^2-y^2}}$, then change to spherical coordinates.

   If you have something like $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^4$ then, because of the constant 4, it would be better to change to cylindrical coordinates.

5. Know how to find the volume of a tetrahedron with given vertices.

6. Know how to calculate the volume of a sphere using cylindrical-spherical coordinates.

   Note: When you are using cylindrical-spherical coordinates, you should have some kind of top and bottom functions.

Change of Variables:

1. Use the given transformation to evaluate the double integral over the region R. (Note: you will have to find the inverse of the transformation (from the xy-plane to the uv-plane) to find your new region S).

2. Evaluate the double integral by making the appropriate change of variables. (Note: you will need to find T (from the uv-plane to the xy-plane) in order to calculate the Jacobian).

Vector Calculus

1. Know how to prove that the gradient vector field $\bigtriangledown f(x,y)$ (or $\bigtriangledown f(x,y,z)$) at the point (x_0,y_0) (or (x_0,y_0,z_0)) is perpendicular to the level curve passing through (x_0,y_0) (or (x_0,y_0,z_0)).

2. **Line Integral**: Evaluate $\int_C f(x,y,z)ds$ over some curve C. Note: this means that you should know how to parametrize curves such as a circle, an ellipse, a line, parabolas, etc...

3. Let C be the curve $r(t) =< x(t), y(t), z(t) >$, $a \le t \le b$. Let $f(x,y,z)$ be a function of 3 variables. Let $F(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$ be a vector field of 3 variables.

   The line integral of f over C is $\displaystyle\int_C f(x,y,z)ds = \int_a^b f(r(t))|r'(t)|dt$. When f=1, we are calculating the length of C.

   The line integral of F over C (=the work or circulation or flow done by F along C) is

   $$\int_C F\cdot Tds = \int_C F\cdot dr = \int_a^b F(r(t))\cdot r'(t)dt = \int_C Pdx + \int_C Qdy + \int_C Rdz.$$

4. Given a vector field F, know how to show if it is conservative or not (F is conservative if and only if $P_y = Q_x$, $P_z = R_x$, $Q_z = R_y$ on a **simply connected region**). If F is conservative, know how to find the potential function f.

   Notice that, even if F is not conservative, it could still have a potential function f. On the other hand, we would not be allowed to use the formula in 6. below.

5. A vector field is conservative

   if and only if $\displaystyle\int_C F\cdot dr = \int_{C'} F\cdot dr = f(r(b)) - f(r(b))$ for any two path C and C' with the same endpoints. (this means that F is independent of path).

   if and only if $\displaystyle\int_\Gamma F\cdot dr = 0$ for any **closed** curve Γ.

6. Know the precise definition of Green's theorem. Know how to use it to find the work done (or the flux) by a vector field along a closed curve and to find the area of a region in the plane.

7. Surface area $\left(A(S) = \int\int_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\ dA\right)$ and surface integral of a function $f(x,y,z)$ over a surface $g(x,y)$ is $\int\int_D f(x,y,g(x,y))\sqrt{(g_x)^2 + (g_y)^2 + 1}\ dA$. For the surface integral, when f=1, then we are actually calculating the surface area of g.

Know how to parametrize: a segment, a circle, an ellipse, a curve y=f(x), and how to change the orientation of these curves. This is really important since all the curves, over which you are integrating either f or F, **must** be oriented.

Make sure you know how to do the homework problems and the problems that I went over in class!

Practice Problems

1. Find the volume of the solid bounded above by $z = 8 - x^2 - y^2$ and below by $z = x^2 + y^2$.

2. Set up only the triple integral for the volume of the solid bounded by $z = 6$, $z = 2y$, $y = x^2$, $y = 2 - x^2$.

3. Express $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} xydzdydx$ as a triple iterated integral in spherical coordinates.

4. (harder) Let S be the solid bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{3x^2 + 3y^2}$. The weight density is d(x, y, z)= z. Find the weight of S.

   (Note: Weight(S) = $\int\int\int (density) dV$.)

5. Express the volume of the solid bounded above by the sphere of radius 5 centered $(0, 0, 0)$ and below by the plane $z = 3$ as a triple integral in spherical coordinates.

6. Find the volume of the solid bounded by the coordinate planes and the plane $x + 2y + 3z = 6$.

7. Let S be the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 1$. The weight density of the solid is $d(x, y, z) = z^2$. Find the weight of S.

8. Find the volume of the solid bounded by $z = 32$ and $z = 2x^2 + 2y^2$.

9. Evaluate the integral $\int_{-2}^{2} \int_{x^2}^{4} e^{y^{3/2}} dydx$.

10. Find the volume of the solid bounded above by $z = 16 - x^2 - y^2$ and below by the xy-plane.

11. Express $\int\int x^2 y dA$ as an iterated integral in polar coordinates where D is the disc of radius 2 centered at (0, -2).

12. Find the volume of the solid bounded by $y = x^2$, $y = 4$, the xy-plane, and $z = x^2$.

13. (harder) Let B be the sphere with equation $x^2 + y^2 + z^2 = 2z$. Describe the volume of B as a triple integral in spherical and cylindrical coordinates.

14. Evaluate the double integral $\int\int_R x^2 y dA$, where R is bounded by $2x - y = 1$, $2x - y = 4$, $x + 3y = 0$, and $x + 3y = 1$.

Good luck!