MATHEMATICAL MODELING: CREATING OPPORTUNITIES FOR PARTICIPATION IN MATHEMATICS

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A Vignette from an Eighth Grade Mathematics Classroom

Aliceson teaches in a small border community at a middle school with a 98% Latino student population. Her class is enrichment, which is an additional required semester class in application of mathematics for all eighth graders. English is the second language for 17 of the 25 students in the class and there is a wide range of levels of reading, writing and mathematical proficiencies. Aliceson formed six heterogeneous groups based on these proficiencies, interests, and personalities. These groups became “families” and have worked together throughout the semester. As a class, they created a set of norms based on respecting all ideas, voices, talents, as well as celebrating differences, that set the boundaries of how members interact and treat each other. Importantly, the students took part in establishing the norms so they took ownership of the ground rules. Aliceson’s role was to create and ensure a safe environment where all students can speak freely, discuss, debate, defend, be heard, take risks, make mistakes, and experience success. During each class period they gather as a whole class and Aliceson launches the task for the day and then the students form their groups by turning their desks towards each other. This has become their routine. Throughout the semester, she has posed modeling tasks for the students to build understanding about how to navigate through and negotiate ways of thinking for solving these open-ended problems. This requires thinking beyond the given information, deciding which information to research, and making assumptions based on their current understanding about the problem context. This particular week, Aliceson posed a problem that was relevant and of high interest to her eighth graders.

The topic of the *Teen TV Viewing* task (see Figure 1) allowed all students in the class to contribute to the conversation about the problem. They all have access to technology at home and in school, and love their "screen time." The launch of the task was in two parts. First they read the July 6, 2015 article on *Teen TV Viewing* and the groups discussed what they believed the author meant by "traditional linear TV." Their conversations included debates about cable, Netflix, watching YouTube videos on phones and "Who are the Nielsens?" Observing this, Aliceson decided to provide some background on the Nielsen Ratings and their purpose.

The following day she began part two of the launch with a whole-class discussion about "traditional television." They created two categories: "free TV" and "cable," and Aliceson used an online resource of the "All-Time 100 TV Shows"[1], and as the shows came up, the students would shout out either "TV" or "cable." Their knowledge of traditional TV programing proved to be greater than she anticipated. Many of these students have family members that live across the border in Mexico, which is only a few miles south of the school. They visit relatives there, many of whom only get TV stations via an antenna. These students also know about TVs that have no remote control. This firsthand experience and knowledge of how people live in remote areas of Mexico is a peculiar feature of working in this border community.

In the next class period Aliceson gave the groups the data and questions in the *Teens TV Viewing* task. Using the stages of the mathematical modeling process that they had been learning

[1] http://time.com/collection/all-time-100-tv-shows/

throughout the semester, they immediately began working systematically through each question of the task. Their mathematical approaches were rich and diverse. Some groups used averages to determine missing data, some looked for patterns, made predictions and validated them. Two groups wrote linear equations to predict the data for 2016 based on trends they viewed over the four previous years. One group intuitively came up with a way of computing relative change. Each group created a presentation that included a graph of the data, the predicted 2016 viewing times, explanations of the percentage change over the previous four years and assumptions about what teens are doing with the time when they are not watching traditional TV. Their assumptions originated from their own experiences: doing homework, participating in hobbies or sports, watching cable or online shows, playing video games, phones, and hanging out with friends.

During their presentations, Aliceson consistently observed well-constructed approaches to mathematics with reasonable outcomes and predictions. Not all work was free of computational errors and some groups were not able to clearly or fluently articulate the process or the group's thinking. She allowed groups to approach all parts of this task however they wished, but they needed to clearly explain, both visually and orally, their approaches and mathematical reasoning. This proved to be challenging during the presentations. Although she encouraged the groups to ask questions and comment about other groups' posters, mathematics and explanations, they were not comfortable critiquing the reasoning of others.

Making a Commitment to Access and Equity

This chapter presents concrete connections between productive beliefs about access and equity in mathematics (NCTM 2014) and classroom activities through the use of mathematical modeling tasks that draw on the students' backgrounds and experiences. Specifically, we highlight three productive beliefs: (1) leveraging students' culture, conditions and language to support and enhance mathematics learning; (2) engaging students with challenging tasks, discourse, and open-ended problem solving to open up greater opportunities for higher-order thinking, and (3) expecting students to make sense of mathematics and persevere in solving challenging problems. Meaningful participation in mathematics involves culturally relevant teaching that utilizes students' backgrounds, knowledge, and experiences (Ladson-Billings 1995) and leads to increased engagement and motivation directly associated with a student's sense of identity (Aguirre, Mayfield-Ingram, and Martin 2013). A commitment to equity in mathematics education "requires that all students have access to high quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (NCTM 2014, p. 59).

Leveraging Students' Background

The vignette describes that when the students discussed the article and the term "traditional linear TV," Aliceson realized the kind of unique understanding they brought to non-cable, non-streaming TV programming, as many of them who visited relatives in small Mexican towns had first-hand experience with television reception by antenna. This fact allowed the task context to connect with students' backgrounds, knowledge, and experiences in a way that generated meaningful engagement with challenging mathematics (see Anhalt, Staats, Cortez & Civil, in press). The vignette describes extensive student participation and Aliceson's facilitating their authentic learning experience in which she values and encourages the use of their cultural backgrounds to bring understanding to the problem, make assumptions, and bring the mathematics they know to the real-life situation.

Engaging Students With Challenging Tasks

The task was developed from a real-world situation described in an article. Consequently, it is not stated as a traditional textbook problem and requires a substantial amount of time and effort by the students to make sense of the situation and the data provided before they can translate this complex reality into a mathematical problem to solve. This high expectation level characterizes Aliceson's class. The vignette shows that this single task linked several mathematics concepts from the curriculum, allowing students to use the mathematics they have learned to address the situation and to experience how different topics come together to solve the problem. The mathematics concepts that emerged in the students' work included averages, pattern detection, linear approximation of data, graphing, conversion from hours:minutes to only minutes, and more.

Expecting Students To Make Sense of Mathematics and Persevere

Getting started on modeling problems can be difficult. Aliceson facilitated the process by using whole-class discussion aimed at making sense of the information presented in the task through the students' experiences. The availability of resources for them to look up information (about the context and about mathematics) and having time for discussions with their peers and the teacher were critical elements for the students to successfully complete the work on this task. Once there was an understanding of the situation and students began to form ideas of how to model it, the students worked with their groups to develop their own approach. Aliceson encouraged students' ideas for solving the problem, knowing that perseverance would be critical to overcome any obstacles along the way.

Advancing Access and Equity

In order to make access a common goal in all mathematics classrooms, teachers must be prepared to seek rich opportunities and experiences for their students. Teachers have the opportunity to adapt the curriculum in ways that connect the mathematics contexts to the students' everyday lived experiences. In this way, access to high quality mathematics can be promoted through carefully selected tasks rooted in contexts that are familiar and relevant to students. Here we describe how teachers can create these opportunities for meaningful participation by more students through the use of modeling tasks.

"Teen TV Viewing Plummets by Fifth Since 2011"

If children are the future, then traditional linear TV's future is anything but bright. Fresh data from Nielsen shows teen TV viewing is down by 20 percent or more since 2011. Nielsen's 2015 Total Audience Report (Q1) clearly demonstrates that traditional linear TV faces serious issues going forward — teen TV viewing is plummeting even as teens watch more video.

How Much TV Are Teens Watching?

The table below gives the amount of time (in hours:minutes) that Americans ages 12-17 watch TV each week. The table shows the times for each quarter of the years since 2011.

	2011	**2012**	**2013**	**2014**	**2015**
Q1: Jan-Mar	24:21	22:14	21:22	21:12	17:52
Q2: Apr-Jun	22:24	21:37	20:39	18:58	16:32
Q3: Jul-Sep	24:11	22:23	21:44	19:12	17:00
Q4: Oct-Dec	22:14	21:28	20:41	18:15	Not available

1. The data for the fourth quarter, Q4, in 2015 was not available. Use some or all of the data in the table to predict the value for Q4 in 2015.
2. The article was written on July 6, 2015 and says that "teen TV viewing is down by 20% or more since 2011." From where does the 20% come?
3. Create a model that explains the rate of decrease in the data and explain if you agree or disagree with the statement the author makes about the decrease of 20%.
4. Use your model to predict the amount of TV viewing in 2016.
5. The article says that the reason teens are watching less TV now than in 2011 is that "Netflix, Hulu, YouTube and other streamers are proving to be more attractive to teen eyeballs." What other reasons can there be? What assumptions is the author making in this statement?

Fig. 1. The teen TV viewing mathematical modeling task

The *Teens TV Viewing* task is based on an online article about the decrease in “traditional TV viewing” by teens from 2011 to 2015. The article provided data and statements that could be analyzed mathematically by students. The task has three important goals: (1) *to fill in* the weekly viewing time in the fourth quarter of 2015, which was missing from the data; (2) *to question* the accuracy of a statement in the article that “teen TV viewing is down by 20% or more since 2011;” and (3) *to predict* TV viewing times in 2016. No specific procedures for doing this were suggested in the task; instead, the students had to decide on an approach to take.

The context was closely connected to Aliceson’s eighth graders, as is likely the case of most teenagers. The context encouraged students to share their stories of watching “traditional linear television” because they had traveled to remote rural areas where no internet or cable was available. Their background experiences allowed them to easily distinguish traditional linear television from online shows. Beyond the context, the task required that the students make assumptions and other choices in order to develop a mathematical problem that addressed the task questions. For instance, to fill in the data for the fourth quarter of 2015, the students had to decide if they should use the data for the previous quarters of 2015 or the data for the fourth quarter of previous years. Either choice could be justified. Having to make and justify choices stimulates sense making. In the case of Aliceson’s students, this manifested itself in searches for patterns, the use of averages, conversations about the data points to generate predictions for 2016, and the type of equations to use. Presenting the solutions publicly helps develop skills that students will need in future school years in all academic areas, and especially in mathematics.

Modeling Features that Play a Role in Providing Access to the Mathematics

Mathematical modeling is a process in which students use mathematics to analyze, understand or predict something associated with an everyday situation. Students use their knowledge of mathematics and of the situation to engage in a cycle of mathematical inquiry. Modeling tasks are usually stated in realistic contexts, tend to be open-ended and allow multiple entry points, have no predetermined algorithm, require assumption-making, and allow approximate solutions (Anhalt 2014). Our approach to increasing access to rigorous mathematics is to introduce challenging open-ended mathematical modeling tasks that require students to make choices (in the form of assumptions, selecting essential variables, etc.) based on their knowledge of the situation and on simplifying the problem to a manageable level. In this way, the students' work is based on their experiences and reflects their background knowledge in mathematics.

The modeling process, as illustrated in Figure 2, is cyclical and the arrows indicate the general path from beginning to end. The process is aligned with the productive belief that "[t]he role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others" (NCTM 2014, p.11).

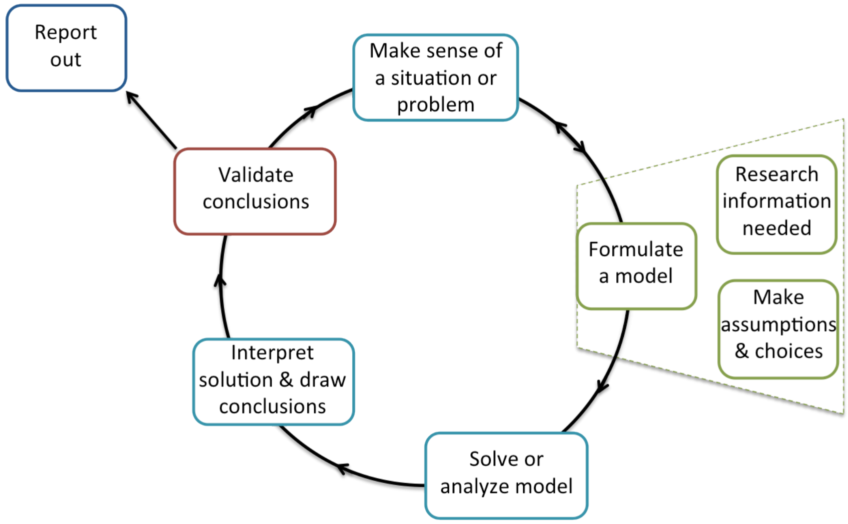

Fig. 2. The mathematical modeling process

Starting at the top of the circle, which is the beginning of the cycle, we illustrate the stages in terms of the *Teens TV Viewing* task. "Make sense of a situation or problem" involves understanding the task and its objective. The situation is often simplified based on existing knowledge and additional research by making assumptions and choices. Some students assumed that the data provided could be approximated with a linear function. Others chose to look for a pattern in the TV viewing times from year to year and detected a pattern. Both approaches were used to predict the TV viewing times in 2016. All assumptions and choices influence the mathematical model that students create.

The process continues with translating the simplified situation into a mathematical model. The model itself consists of mathematical equations, expressions or algorithms that represent the original situation. The linear equation relating TV viewing times to the year is an example of a mathematical model (Figure 3a). A different model is the formula for the missing data in the fourth quarter of 2015 as the average of the previous quarters (Figure 3b). Solve or analyze the model is a mathematical problem-solving step where we may consider various representations and strategies in order to solve the mathematical problem described in the model. This solution needs to be interpreted in the context of the situation. For instance, when the data were approximated by a line with slope -1.5, this was correctly interpreted as a decrease of 1 minute and 30 seconds per year in TV viewing times. Any conclusions drawn from the model must be validated. If the conclusions are not acceptable, modifications to the choices, assumptions or model need to be made and start through the process again. If the conclusions are acceptable, the solution should be reported out including the assumptions under which the conclusions are valid.

Using Modeling Tasks to Promote Productive Beliefs About Access and Equity

Providing students with opportunities to utilize their experiential knowledge in mathematics allows them to participate in the intellectual work of modeling. Because modeling tasks are connected to a context, they can be adapted to students' interests to draw on background knowledge. By doing so, teachers are able to leverage students' intellectual resources as they make sense of the situation, make choices, establish assumptions, and define variables that impact the mathematical model. Mathematical modeling is an important component of the mathematics curriculum that can readily (a) incorporate students' culture, conditions, and language; (b) engage students with challenging tasks; and (c) allow students to make sense of mathematics and persevere in solving challenging problems. Below, we elaborate on these productive beliefs in connection to mathematical modeling.

Leverage students' culture, conditions, and language. Mathematical modeling tasks have the potential to leverage students' culture through decisions and assumptions for solving the problem. The open-endedness of the tasks caters the opportunity for students to talk about missing information that they can provide from their background knowledge. During the launch of the *Teens TV Viewing* task, the students used their background knowledge to distinguish between traditional TV and streaming sites or cable to offer possible reasons for the decline in traditional TV watching times over the years. The visits to rural Mexico by some of the students were a significant part of their experiential knowledge and became important to make sense of the task. Through group discussions, the students negotiated assumptions and approaches for solving the problem and ultimately create their models.

Engage students with challenging tasks. The modeling process is itself challenging, yet the opportunity for students to draw on their experiences for solving modeling tasks is key to providing access and participating in the rigor of mathematics. For example, in the *Teens TV Viewing* task, the students applied knowledge of mathematics concepts and skills they knew in a familiar context, thereby, giving them the opportunity to deepen their understanding of the mathematics they were using. In addition to applying problem solving skills, decontextualizing the situation, interpreting the results, validating the solutions, and sharing ideas, the problem can be solved using a combination of several concepts in mathematics, such as ratios, decimals, integers, fractions, linear equations, approximations, averages, and conversions of minutes and hours to a single unit of time. Making and justifying choices through the end of the process is part of what makes mathematical modeling challenging. When students gain an understanding of the available options and decisions they can make within a modeling task, the choices they make based on their knowledge offer them confidence and ownership of the problem (Anhalt 2014).

Expect students to make sense of mathematics and persevere in solving challenging problems. Working through the modeling process forces the students to think and rethink assumptions to improve the model as they continue to gain a deeper understanding of the situation. The *Teens TV Viewing* task required several class periods for the students to arrive at the models and solutions. The students put forth sustained effort as they worked through the task taking different approaches in the mathematics. They chose their approach based on a collective group understanding of the problem and diligently prepared their posters to share their work.

Students' Engagement and Work in Mathematics

Figure 3 shows two samples of the work of eighth graders on the *Teens TV Viewing* task. The top row shows the poster created by students that realized that data given in hours and minutes (e.g. 24:21) needed conversion to a single unit of time; they chose to use minutes. The bottom left side of the poster shows their conversion of 24:21 to 1461 minutes and 17:52 to 1072 minutes. Their relative difference, (1461-1072)/1461 = 0.28 was correctly interpreted as a 28% decrease in teen TV viewing between 2011 and 2015. The right side of the poster shows the group's linear approximation to the data. Figure 3b shows the poster of the group that incorrectly thought that basic operations could be carried out without conversion to a single unit of time until a final answer was reached. Their poster shows that they computed the average of 17:52, 16:32 and 17:00 as (50:84)/3 = 16:94 treating the symbol "**:**" as a decimal point. They then wrote 16:94 as 17:34, showing awareness that a conversion was needed at the end. Here, the modeling approach (averaging) was reasonable but there was a conceptual error computing with the data. This activity was informative to Aliceson as it revealed concepts and skills that needed attention.

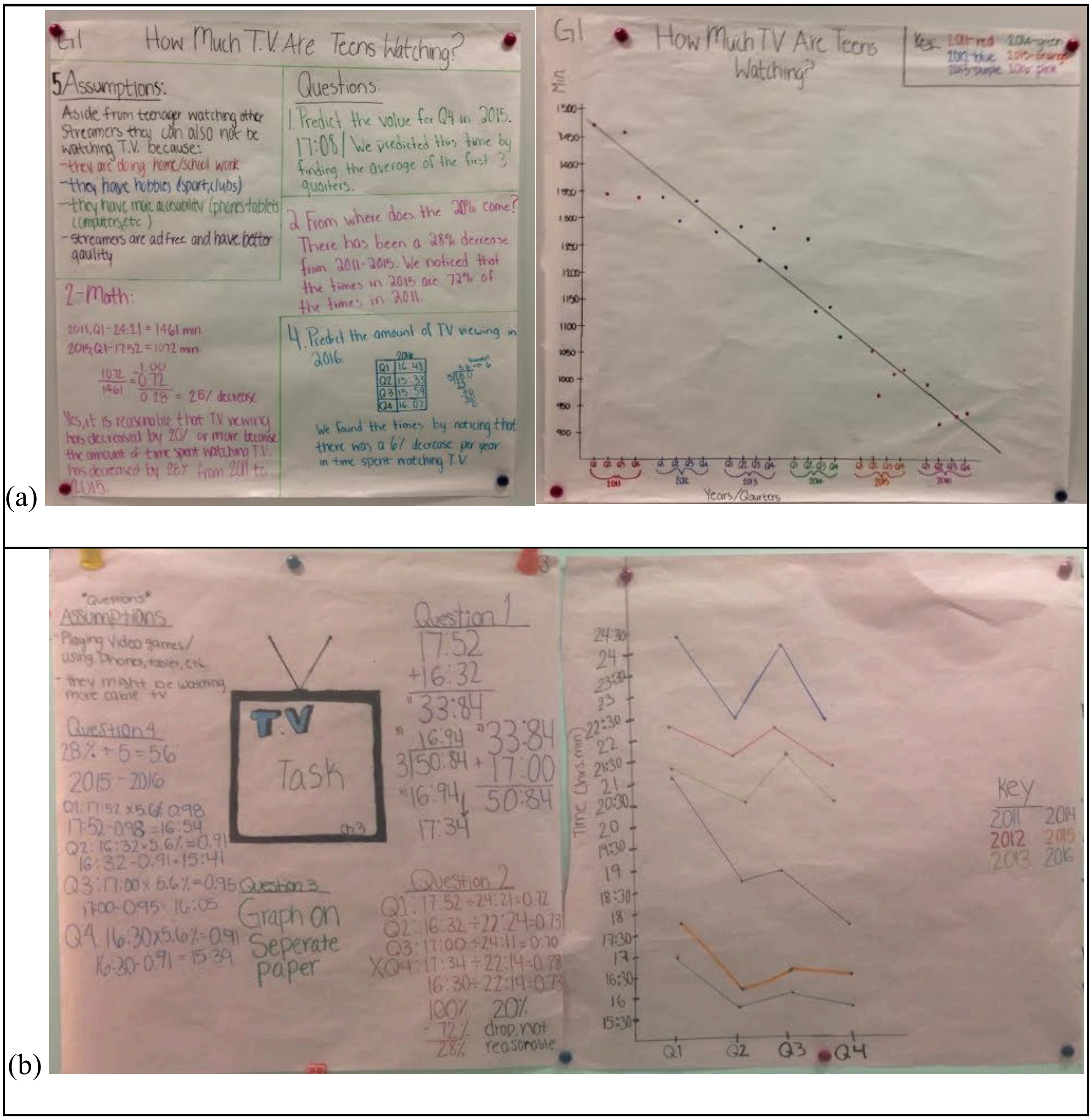

Fig. 3. Samples of eighth graders' work: (a) linear approximation; (b) averaging data

Throughout the semester, the students had developed experience in recognizing when assumptions are necessary to formulate a model as this is a critical component of modeling.

Figure 3 shows the students' assumptions as part of the communication of their work, their solution, and justification for the assumptions. Expecting students to communicate solutions and conclusions provided opportunities for them to express themselves mathematically, using formal and informal language. Aliceson noted that it was challenging to find ways to elicit deep descriptions, both orally and in writing, of her students' mathematical reasoning. To address this challenge, following the presentations, Aliceson asked the students to discuss and then write a description of how they approached the task, and to think of other examples of tables, charts and equations they could add to their existing work. This was the first time they were asked to go through this kind of reflection and writing exercise, and although it proved to be difficult, the students gained more experience communicating mathematical reasoning beyond procedure.

Reflecting and Taking Action

During this project, Aliceson participated in a professional development project, Mathematical Modeling in the Middle Grades (M^3) with teachers from fourth to eighth grades in two rural school districts near the U.S.-Mexico border. The project focused on introducing the mathematical modeling process and having the teachers experience hands-on problems that are relevant to their students and communities. The teachers modified or created tasks to implement with their students in a format similar to what they experienced in the project: (1) launching the problem by introducing a context, allowing for discussion to elicit students' individual knowledge of the situation, and giving them autonomy to make assumptions based on their experiential knowledge; (2) exploring mathematical ways to create a model that follows their assumptions, interpreting their results and validating their conclusions; and (3) creating posters to report their models and the conditions under which they are valid.

An important aspect of the M^3 project was to leverage the communities' cultural backgrounds through modeling tasks that were relevant to the area. The tasks included border-crossing data, produce imported to the U.S., community food banks, internet and cable access, and water conservation among others. While these issues may be relevant to various communities across the U.S., the background knowledge that teachers and students bring to the specific context contribute in significant ways to the assumptions and choices made that directly impact the mathematical models.

Aliceson implemented modeling problems in her class throughout the semester. She created a rich environment for her students to take ownership of the mathematics they used in solving the problems, to gain confidence in participating in class discussions, and to create posters to share their work. Aliceson shared that the project has given her the tools to teach problem solving and modeling in meaningful ways. Having groups work collaboratively on open-ended tasks encouraged her students to think deeply, create relevant assumptions, have mathematical conversations, do research, listen, create models, draw conclusions, and share their solutions. Engaging all students with mathematics has become a priority in Aliceson's classroom.

Aliceson's Reflection

I found mathematical modeling to be a tremendous learning experience for both my students and me. The students enjoyed the *Teens TV Viewing* task because of the topic, and the mathematics had different possible approaches. They felt less intimidated and less pressured to get the "right answer" and more freedom in the process of pursuing a mathematical model. They enjoyed creating the posters for communicating their mathematical thinking and solutions to the modeling tasks. Posters will remain a part of my teaching so students can showcase their thinking and learning. As I observed my students working, I noticed that each one has a talent, knowledge, or idea to contribute. Posing mathematical modeling tasks helped me create a safe classroom environment where students can take the necessary risks to experience both failure and success in positive ways, and this process has allowed me to comfortably move from a teacher-centered classroom, such as being "the sage on the stage" to the student-centered classroom which situates me as "the guide on the side." I enjoyed the interactions, the contributions, and ultimately, their learning mathematics through modeling.

In the *Teens TV Viewing* task I saw an immediate need for further instruction in the areas of percent of change, converting between units of measurement related to time, constructing viable arguments and critiquing the reasoning of others, and communicating results. I created subsequent lessons that involved the use of decimals, conversions, graphing, and variables. In future modeling tasks, the writing component that asks students to reflect on the approaches they used and their solutions to the task, will remain a requirement as it is a vital part of students' development in communicating mathematically.

To improve on providing students an opportunity to critique the reasoning of others and learn to ask for clarification, I plan to introduce the class to a silent "gallery walk" of all the posters before students give their presentations. I will require students to take notes and discuss each poster and prepare questions to ask during the presentations. I predict that this will better prepare them for listening to each other.

Recommendations for Using Mathematical Modeling to Promote Access

Mathematical modeling can be an empowering experience for all students as their contributions can be meaningful. Students need time and experience to become proficient in learning how the choices they make impact the mathematics of the solution and in using the mathematics they know to build new mathematical knowledge. In selecting or creating problems, it is useful to know the students' interests, families, communities, recreational activities outside of school, or travel experiences so that when topics arise in the context of the problems, the teacher can leverage the students' experiential knowledge. Aliceson surveys her students early in the semester to collect information on their interests and backgrounds so that she may use that information in problem contexts.

Another recommendation for promoting access through mathematical modeling is to allow students time for structured discussion in small groups to give them opportunities to find out what their peers know about a given situation. This encourages students to be curious about a variety of topics and do necessary research to inform their assumptions and mathematical models. In addition, teachers can establish a classroom environment where students feel safe to take risks, and where failure is viewed as a learning opportunity.

Mathematical modeling tasks provide access opportunities for students to engage with mathematically challenging tasks, discourse, and open-ended problem solving based on real-life contexts that are meaningful to students. Modeling supported by scaffolding strategies (see

Anhalt 2014) within familiar community practices (see LópezLeiva 2014) is empowering for students and builds their confidence to use mathematics to explore everyday situations. It is reasonable to set high levels of expectations for students when support and encouragement are provided as students gain confidence in understanding their roles and accountability when engaging in the mathematical modeling process. Interweaving the students' culture, conditions, and language backgrounds within challenging and relevant mathematical modeling tasks has promising outcomes for students.

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