# Johns Hopkins University, Department of Mathematics 

### 110.105 Intro to Calculus - Fall 2013

## Final Exam

Instructions: This exam has 14 pages and is out of a total of 200 points. No calculators, books or notes allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. If you use the extra paper, be sure to clearly label which problem it belongs to and attach it to the exam. You have 180 minutes.

## Academic Honesty Certification

I certify that I have taken this exam with out the aid of unauthorized people or objects.

Signature: $\qquad$ Date: $\qquad$

Name: $\qquad$

Some identities:

$$
\begin{gathered}
\cos (\alpha+\beta)=\cos (\alpha) \cos (\beta)-\sin (\alpha) \sin (\beta) \\
\cos (\alpha-\beta)=\cos (\alpha) \cos (\beta)+\sin (\alpha) \sin (\beta) \\
\sin (\alpha+\beta)=\sin (\alpha) \cos (\beta)+\cos (\alpha) \sin (\beta) \\
\sin (\alpha-\beta)=\sin (\alpha) \cos (\beta)-\cos (\alpha) \sin (\beta) \\
\cos (2 x)=1-2 \sin ^{2}(x)=\cos ^{2} x-\sin ^{2} x \\
\sin (2 x)=2 \sin x \cos x
\end{gathered}
$$

1. (20 total points)
(a) Evaluate

$$
\left(-\frac{125}{8}\right)^{2 / 3}
$$

(b) Simplify

$$
\sqrt[3]{-27 x^{6}}
$$

(c) Factor

$$
x^{2}-13
$$

(d) Find the domain of $f(x)=x^{-1 / 2}$
(e) Solve the inequality

$$
-2 x+1 \leq 3
$$

2. (15 points) Graph at least two cycles of $f(x)=-\cos \left(\frac{1}{3} x\right)$ (Hint: Start by finding the period)
3. (20 total points) Consider the polynomial

$$
P(x)=-2 x^{3}+4 x^{2}+8 x-16
$$

(a) Factor $P(x)$ completely. (Hint: $x=2$ is a zero of $P(x)$ )
(b) Sketch a rough graph of $P(x)$. Be sure to clearly label the $x$ and $y$-intercepts and think about the behavior at the $x$-intercepts and at the ends.
4. (15 points) Verify the following trigonometric identity. Be sure to show all work and clearly justify each step (for example, if you use a Pythagorean identity, say so).

$$
\cos ^{4}(x)-\sin ^{4}(x)=\cos (2 x)
$$

(Hint: Start by applying the difference of squares formula to the left hand side)
5. (20 total points)
(a) Evaluate

$$
\log _{6}(4)+\log _{6}(9)
$$

(b) Simplify

$$
\ln \left(\ln \left(e^{e^{3 x}}\right)\right)
$$

(c) Solve the equation

$$
2^{3 x}=4^{-x+3}
$$

(d) Solve the equation

$$
\log _{x}(4)=\frac{1}{2}
$$

6. (15 points) Sketch a rough graph of the rational function below. Label all asymptotes and $x$ and $y$-intercepts.

$$
f(x)=\frac{x}{x+1}
$$

7. (20 total points)
(a) Given that $\cos \left(37^{\circ}\right) \approx \frac{4}{5}$, find $\sin \left(74^{\circ}\right)$. (Hint: you will need to find $\sin \left(37^{\circ}\right)$ first)
(b) Find all solutions to the equation $\cos (x)=-\frac{\sqrt{3}}{2}$.
(c) Find $\arcsin (\cos (\pi / 3))$.
8. (20 total points) Consider the complex number $z$ with direction $\pi / 4$ radians and magnitude $\sqrt{2}$
(a) Write $z$ in standard form.
(b) Find $z^{6}$. This will be much easier if you write $z$ in trigonometric form first. You may give your answer in standard or trigonometric form.
9. (15 points) Graph the following equation in polar coordinates

$$
r=1-\cos (\theta)
$$

10. (20 points) Solve the following system of equations by substitution:

$$
\left\{\begin{array}{l}
3 x+5 y=-2 \\
7 x-2 y=9
\end{array}\right.
$$

11. (20 points) Solve the SAS triangle with $\alpha=45^{\circ}, c=1+\sqrt{3}$, and $b=\sqrt{2}$.

Start by finding $a$, then find $\beta$, then find $\gamma$ (in that order). If you do everything correctly, you'll be able to solve the triangle without using a calculator.

Extra paper!

More paper!

