Completing the square (handout).

Let's look at an example:

$$x^2 + 4x = 5$$

we would like to write the left hand side as a square, that is something like $(x+a)^2$ or $(x-a)^2$.

but $(x \pm a)^2 = x^2 \pm 2ax + a^2$, so we might need to add something to this equation:

Let's add 4 to both sides! Then:

$$x^2 + 4x + 4 = 5 + 4 = 9$$

Now we can write $x^2 + 4x + 4$ as $(x+2)^2$!

Sometimes it is harder to see

Let's look at another example:

$$x^2 + 3x = 5$$

If we add $\frac{9}{4}$ then we get:

$$x^2 + 3x + \frac{9}{4} = 5 + \frac{9}{4}.$$

But $\left(x + \frac{3}{2}\right)^2 = x^2 + \frac{3}{2} \cdot x \cdot 2 + \left(\frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}$.

Thus $\left(x + \frac{3}{2}\right)^2 = 5 + \frac{9}{4}$.

In general how do we figure out what to add?

If your equation is:

$x^2 + ax = b$ then write it as:

$x^2 + 2 \cdot \frac{a}{2} \cdot x = b.$

then add $\left(\frac{a}{2}\right)^2$ to both sides to get:

$$x^2 + \frac{2}{2}ax + \left(\frac{a}{2}\right)^2 = b + \left(\frac{a}{2}\right)^2$$

$$\left(x + \frac{a}{2}\right)^2 = b + \left(\frac{a}{2}\right)^2.$$