Johns Hopkins University, Department of Mathematics

Introduction to Calculus - Fall 2014

Midterm 1

Instructions: This exam has 11 pages. No calculators, books or notes allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. Be sure to clearly label each problem and attach them to the exam. You have 50 MINUTES.

Name: Kalina

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: _____ **Date:** _____

PLEASE DO NOT WRITE ON THIS TABLE !!

Problem	Score	Points for the Problem
1		50
2		20
3		40
4		20
5		20
TOTAL		150

Question 1. [50 points] Consider the function $f(x) = \sqrt{x+1} - 3$.

(a) [10 points] Find the domain and the range of $f(x)$.

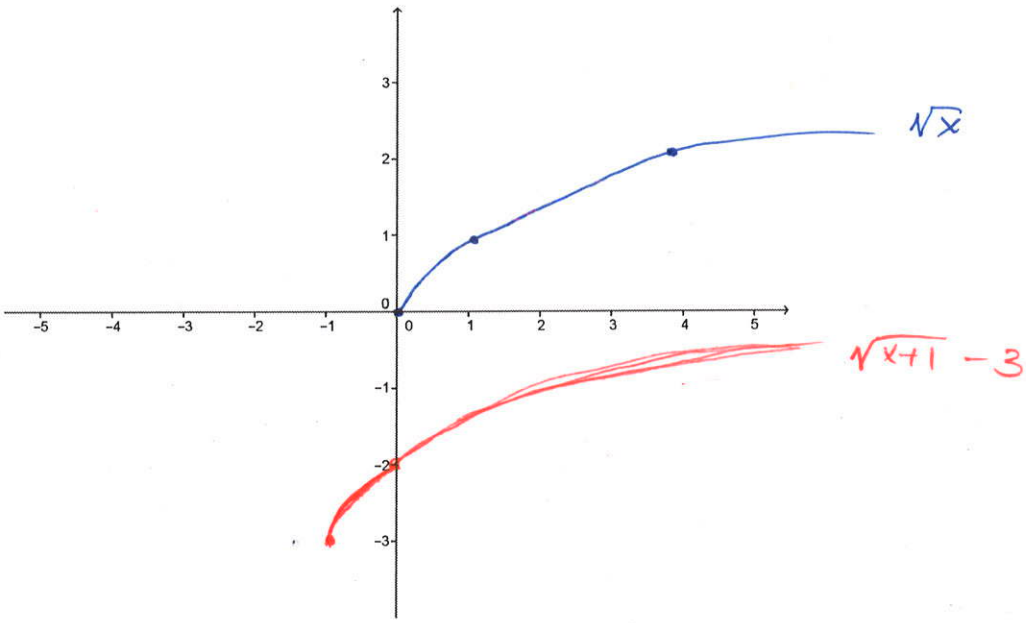
domain $x+1 \geq 0 \Rightarrow x \geq -1$. $(-1, \infty)$

range: $f(x) = y \geq -3$ $(-3, \infty)$.

(the smallest value of $\sqrt{x+1}$ is 0, when $x=-1$
& $\sqrt{x+1}$ is ≥ 0)

that's why $f(x) \geq -3$.

(b) [15 points] Sketch below the graph of $\sqrt{x}$ and the graph of $f(x) = \sqrt{x+1} - 3$. Make sure you label your graphs. How do you obtain one from the other - do you translate, reflect, scale - in which direction and by how much. Explain!



x	0	1	4	9
$\sqrt{x}$	0	1	2	3

graph of $\sqrt{x+1}$ is shifted to the left 1 from the graph of $\sqrt{x}$

— " — $\sqrt{x+1} - 3$ is shifted down 3 from the graph of $\sqrt{x+1}$

(c) [5 points] Is f one-to-one? Explain.

yes - horiz. line test.

(d) [10 points] Find the inverse of f, i.e. f^{-1}.

$y = \sqrt{x+1} - 3$

switch x & y & solve

$x = \sqrt{y+1} - 3$

$x+3 = \sqrt{y+1}$ (raise both to second power) ie. square both sides

$(x+3)^2 = y+1.$

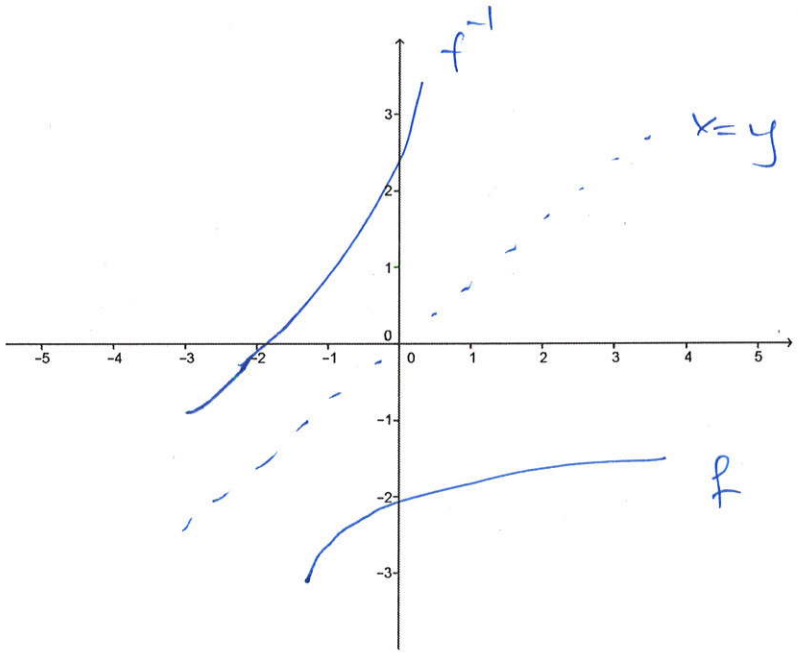
$y = (x+3)^2 - 1 = x^2 + 6x + 8. = f^{-1}$

(e) [5 points] What is the domain and range of f^{-1}.

domain f^{-1} = range f

range f^{-1} = domain f

(f) [5 points] Sketch the graph of f^{-1}. (Hint: Recall that the graphs of f and f^{-1} are related.)



f & f^{-1} have graphs that are reflect. of each other about the line $x=y$.

Question 2. [20 points] Factor the polynomial completely $f(x) = x^3 - 2x^2 + 3x - 6$, that is find all real and complex roots! (Hint: Check which are the rational zeros first. Then use the Decartes's rule of signs to eliminate some of them. Use the quadratic formula to factor quadratic polynomials.) Explain your work!

$$x^3 - 2x^2 + 3x - 6 = 0.$$

possible rational roots:

$$\pm 6, \pm 3, \pm 2, \pm 1.$$

Decartes's rule of signs:

$$f(x) = x^3 - 2x^2 + 3x - 6$$

$$+ \quad - \quad + \quad -$$

3 or 1 real (positive) roots.

$$f(-x) = -x^3 - 2x^2 - 3x - 6$$

0 negative real roots.

$\Rightarrow$ try $+6, +3, +2, +1$.

$f(2) = 0$.

2	1	−2	3	−6
		2	0	6
	1	0	3	0

$f = (x-2)(x^2+3)$

& $x^2 + 3 = (x + i\sqrt{3})(x - i\sqrt{3})$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2b}$

Question 3. [40 points] Sketch the graph of the polynomial function $f(x) = x^3 - 5x^2 + 7x - 3$.
The following steps will lead you through the process:

(a) [5 points] Is the function symmetric in any way (is it odd, even quadratic function)? Why? Why not?

$f(-x) = -x^3 - 5x^2 - 7x - 3$

$-f(x) = -x^3 + 5x^2 - 7x + 3$

$f(-x) \neq f(x) \Rightarrow$ not even

$-f(x) \neq f(x) \Rightarrow$ not odd.

& clearly not of quadratic type.

(b) [15 points] What are the x-intercepts?

$f = x^3 - 5x^2 + 7x - 3$ possible roots $\pm 3, \pm 1$.

$f(1) = 0$.

$$\begin{array}{c|cccc} 1 & 1 & -5 & 7 & -3 \\ & & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$\Rightarrow f = (x-1)(\underbrace{x^2 - 4x + 3}_{(x-1)(x-3)})$.

$f(x) = (x-1)^2(x-3)$

the x-intercepts: $(1,0)$ & $(3,0)$.

(c) [5 points] How does the function behave at the x-intercepts? (crosses the x-axis, touches the x-axis)? Why?

$x \longrightarrow \infty$

$f(x) = y \longrightarrow \infty$

$x \longrightarrow -\infty$

$f(x) = y \longrightarrow -\infty$

(d) [5 points] What happens when $x \to \infty$ and when $x \to -\infty$?

at $x = 1$ touches the x-axis (root has multip. 2)

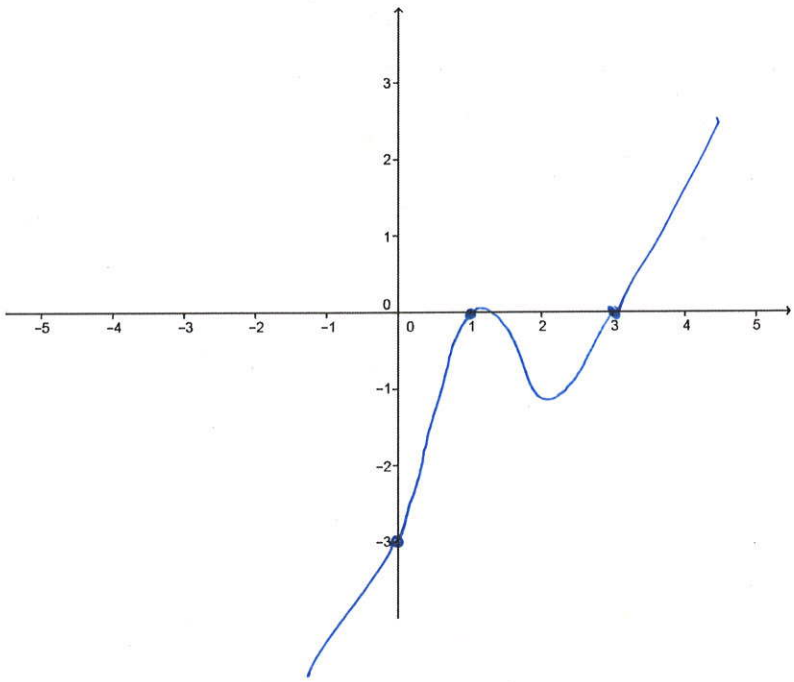
at $x = 3$ intersects the x-axis (root has multip. 1)

(e) [5 points] Find the y-intercept(s)?

$x=0$, $f(0)=-3$

the y-intercept is $(0,-3)$.

(f) [5 points] Now sketch the graph of $f(x)$.



Question 4. [20 points] Consider the rational function $R(x) = \frac{2x^2-x}{x-1}$.

(a) [5 points] What is the domain of $R(x)$? (Hint: Recall how to find the domain and range of a quotient of two functions $f(x)/g(x)$.)

the domain is all $x \neq 1$.

(if $f(x) = 2x^2 - x$

$g(x) = x - 1$.

recall that the domain of $f(x)/g(x)$ is the intersection of the domains of f & g minus the roots of g. But since f & g are polynomials their domains are $\mathbb{R}$.).

(b) [BONUS 5 points] What is the range of $R(x)$? Explain!

the range is $\mathbb{R}$.

(one way to see this is to look at the graph of $R(x)$).

(c) [10 points] Find the asymptotes if there are any (horizontal, vertical, slant). Explain your work!

$x=1$ is a vertical asymptote.

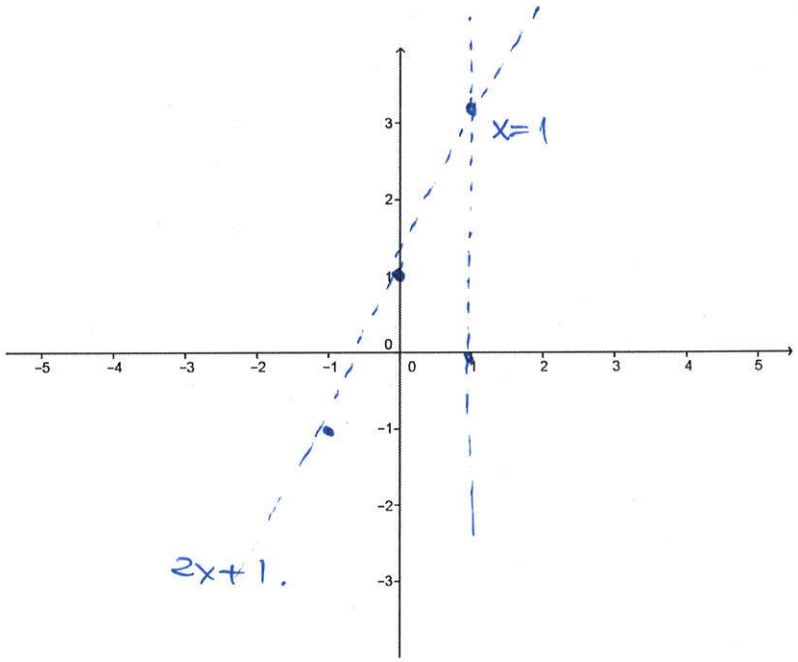
$$\frac{2x^2-x}{x-1} = (2x+1) + \frac{1}{x-1}$$

$$\begin{array}{c|ccc} 1 & 2 & -1 & 0 \\ & & 2 & 1 \\ \hline & 2 & 1 & 1 \end{array}$$

as $x \to \pm\infty$ $y \to 2x+1$

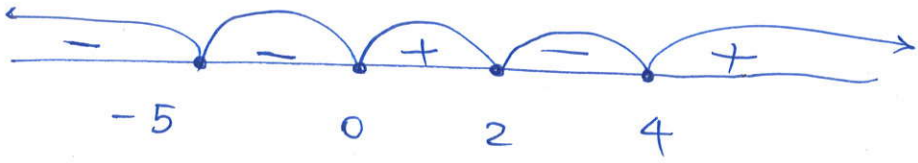
$\Rightarrow$ slant asymptote $2x+1$.

(d) [5 points] Sketch the asymptotes you found.



Question 5. [20 points] Solve the inequality $x(x-4)(x-2)(x+5)^2 \leq 0$. Write the answer as an interval. Explain your work!

0 4 2 -5



$f(x) = x(x-4)(x-2)(x+5)^2$

for $x > 4$ (check each factor)

$+ \quad + \quad + \quad + \quad \Rightarrow \quad f(x) > 0.$

for $x \in (2,4)$ (check each factor)

$+ \quad - \quad + \quad + \quad \Rightarrow \quad f(x) < 0.$

for $x \in (0,2)$

$+ \quad - \quad - \quad + \quad \Rightarrow \quad f(x) > 0.$

for $x \in (-5,0)$

$- \quad - \quad - \quad + \quad \Rightarrow \quad f(x) < 0.$

for $x < -5$

$- \quad - \quad - \quad + \quad \Rightarrow \quad f(x) < 0.$

$\Rightarrow f(x) \leq 0$ for $x \in (-\infty, 0] \cup [2,4]$.