Johns Hopkins University, Department of Mathematics

Introduction to Calculus - Fall 2014

Midterm 2

Instructions: This exam has 6 pages. No calculators, books or notes allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. Be sure to clearly label each problem and attach them to the exam. You have 50 MINUTES.

Name: Kalina

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________ **Date:**__________

PLEASE DO NOT WRITE ON THIS TABLE !!

Problem	Score	Points for the Problem
1		18
2		25
3		12
4		22
5		23
TOTAL		100

Some identities that you might find useful:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$
$$\cos(2x) = 1 - 2\sin^2(x) = \cos^2 x - \sin^2 x$$
$$\sin(2x) = 2\sin x\cos x$$
$$\tan^2(x) + 1 = sec^2(x)$$

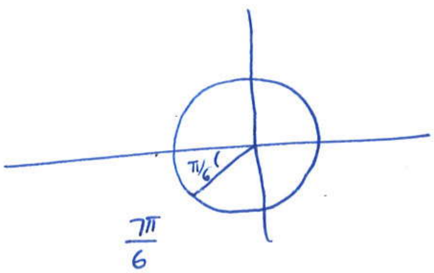
Question 1. [18 points] Evaluate the following expressions. Put your answer in the simplest terms possible.

(a) [6 points] $\sin(\frac{7\pi}{6})$

$\frac{7\pi}{6}$ is in the 3rd quadrant $\Rightarrow \sin\frac{7\pi}{6} < 0$

reference angle is $\pi/6$

$\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$.



(b) [6 points] $\cos(\frac{\pi}{12})$, Hint: $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cdot\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cdot\sin\frac{\pi}{4}$

$= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

(c) [6 points] $\cos^{-1}(0)$, Hint: Keep in mind where $\cos^{-1}(x)$ is defined.

$\cos^{-1}(x)$ is defined on $[0, \pi]$.

$\cos^{-1}(0) = \frac{\pi}{2}$, b/c $\cos\frac{\pi}{2} = 0$, $\frac{\pi}{2} \in [0, \pi]$.

Question 2. [25 points] Find *all* solutions to the following equations.

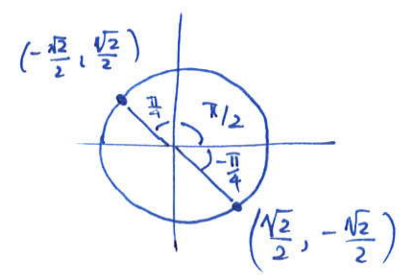
(a) [5 points] $\log(x-3)=4$

$\log(x-3)=4$

$x-3=10^4$

$x=10^4+3=10\,003$

(b) [8 points] $\tan(x)=-1$



$\tan x=-1$ at $\sin x=-\cos x$.

if $x\in[0,2\pi]$

we have two such points

$\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$ & $-\frac{\pi}{4}$ or $2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$

all solutions are:

$$\begin{cases}\frac{3\pi}{4}+2k\pi\\[2ex]\frac{7\pi}{4}+2k\pi\end{cases}$$

(c) [12 points] $6^x = 2^{x-1}$

(we can take $\log_a$ - on both sides. I choose $a=e$.)

+ 5 $x \ln 6 = (x-1) \ln 2$

+ 1 $x \ln 6 = x \ln 2 - \ln 2$

+ 2 $x(\ln 6 - \ln 2) = -\ln 2.$

+ 2 $x \ln 3 = -\ln 2$

+ 2 $x = \dfrac{-\ln 2}{\ln 3}$

Question 3. [12 points] Verify the following identity. Be sure to show all work and clearly justify each step (for example, if you use a Pythagorean identity, say so).

$$\underbrace{\frac{1}{1+\cos(-x)} + \frac{1}{1-\cos(x)}}_{lhs} = 2\csc^2 x.$$

$$lhs = \frac{1}{1+\cos x} + \frac{1}{1-\cos x} = \frac{1-\cos x + 1 + \cos x}{(1+\cos x)(1-\cos x)} = \frac{2}{1-\cos^2 x}$$

$$= \frac{2}{\sin^2 x} = 2\csc^2 x.$$

Question 4. [22 points] Graph a cycle of the following trig function. Be sure to label the axes. (Hint: you might want to sketch $\sin(x)$ first, then transform its graph to get the graph of the function we are looking for - by changing the amplitude and the period and translating vertically/horizontally).

$$f(x) = 2\sin(2x) + 1.$$

"Some things aren't on the Cartesian plane" Topher Brink.

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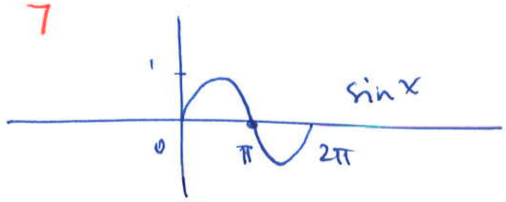

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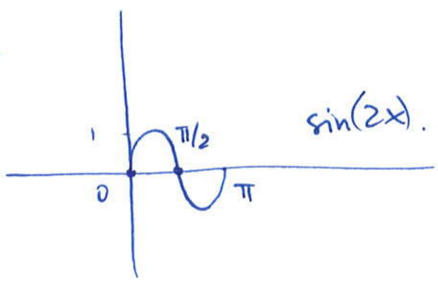
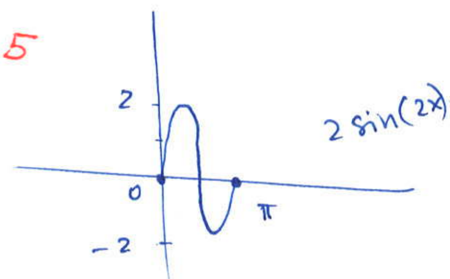


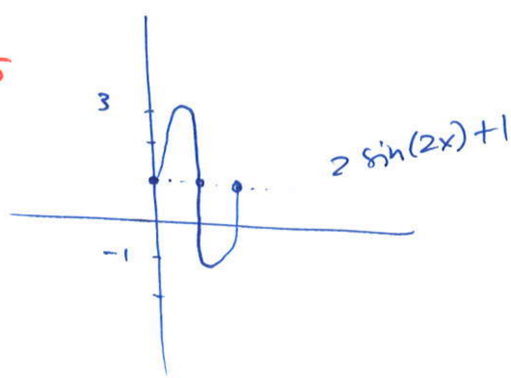
change period.

5



change amplitude.

5



shift up one.

Question 5. [23 points] Consider a SSA triangle with $\alpha = 30°, a = 3$ and $b = 5$. You may use the fact that $\sin(56.44) = \sin(123.56) = 5/6$, $\sin(93.56) \approx 1$ and $\sin(26.44) = 0.445$.

(a) [5 points] Determine how many such triangles there are by finding h.

$h = \sin\alpha \cdot b = \frac{1}{2} \cdot 5 = 2.5.$

$b > a > h \Rightarrow$ 2 triangles. (SSA case 3).

(b) [13 points] Solve all possible triangles.

+4 $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \Rightarrow \frac{\frac{1}{2}}{3} = \frac{\sin\beta}{5} \Rightarrow \sin\beta = \frac{5}{6} \overset{+2}{\Rightarrow} \beta = 56.44$ or $\beta = 123.56.$

+3 $\gamma = 180° - 30° - \beta = 150° - 56.44$ or $= 93.56$
$150° - 123.56 = 26.44.$

+4 $\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} \Rightarrow \frac{1}{6} = \frac{\overbrace{\sin(93.56)}^{1}}{c}$ or $\frac{1}{6} = \frac{\overbrace{\sin(26.44)}^{0.44.}}{\cancel{0.445}\, c}$

$c = 6$ or $c = 6.(0.44).$

(c) [5 points] Find the area of the triangle(s).

area $= \frac{1}{2}\, ab \sin\gamma = \frac{1}{2}\, 3 \cdot 5 \cdot 1$ or $\frac{1}{2} \cdot 3 \cdot 5 \cdot 0.44.$

$= 7.5$ $\quad \frac{15}{2} \cdot (0.44) = 7.5 \cdot (0.44)$