# Johns Hopkins University, Department of Mathematics

## Introduction to Calculus - Fall 2014

## Exam 1

**Instructions:** This exam has 6 pages. No calculators, books or notes allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. Be sure to clearly label each problem and attach them to the exam. You have 30 MINUTES.

Name: _Kalina_____

### Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: _____ Date:_____

## PLEASE DO NOT WRITE ON THIS TABLE !!

| Problem | Score | Points for the Problem |
|---------|-------|------------------------|
| 1 | | 21 |
| 2 | | 21 |
| 3 | | 12 |
| 4 | | 12 |
| 5 | | 21 |
| 6 | | 7 |
| 7 | | 6 |
| TOTAL | | 100 |

**Question 1.** [21 points] Simplify the following expressions.

    **(a)** [10 points]

$$-\frac{\overset{5}{15}x^{8/3}}{\underset{6}{18}x^{-1/3}} = -\frac{5}{6}x^{8/3+1/3} = -\frac{5}{6}x^{9/3} = -\frac{5}{6}x^3$$

Note!

$$x^{8/3} \cdot x^{-1/3} = x^{8/3-1/3} = x^{7/3}$$

but

$$\frac{x^{8/3}}{x^{-1/3}} = x^{8/3} \cdot x^{1/3} = x^{8/3+1/3} = x^{9/3} = x^3$$

    **(b)** [11 points]

$$\frac{\frac{x-3}{y}}{\frac{1-x}{(x+2)y}} = \frac{x-3}{y} \div \frac{1-x}{(x+2)y} =$$

$$= \frac{x-3}{y} \cdot \frac{(x+2)y}{1-x} = \frac{x^2-x-6}{1-x}$$

PLEASE SHOW YOUR WORK OR EXPLAIN YOUR REASONS

**Question 2.** [21 points] Solve the following inequalities. Write the solution as an interval.

    **(a)** [9 points] $3x + 2 > x$.

$$3x - x > -2$$
$$2x > -2$$
$$x > -1$$

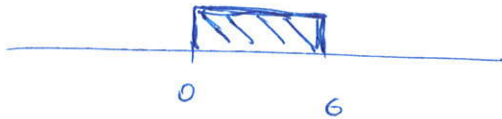$$x \in (-1, \infty).$$

    **(b)** [12 points] $|x - 3| + 2 \leq 5$.

$$|x - 3| \leq 3$$

$$-3 \leq x - 3 \leq 3$$

$$-3 \leq x - 3 \qquad \& \qquad x - 3 \leq 3$$
$$\qquad\qquad\qquad\qquad\qquad x \leq 6$$
$$0 \leq x$$

$$x \in [0, 6].$$

**Question 3.** [12 points] Solve the following quadratic equation

$$x^2 + 5x + 6 = 0.$$  $(x+2)(x+3) = 0.$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4 \cdot 6}}{2.}$$

$$= \frac{-5 \pm 1}{2} \Rightarrow \boxed{\begin{array}{l} x_1 = -3 \\ x_2 = -2 \end{array}}$$

**Question 4.** [12 points] Find the domain of the rational expression

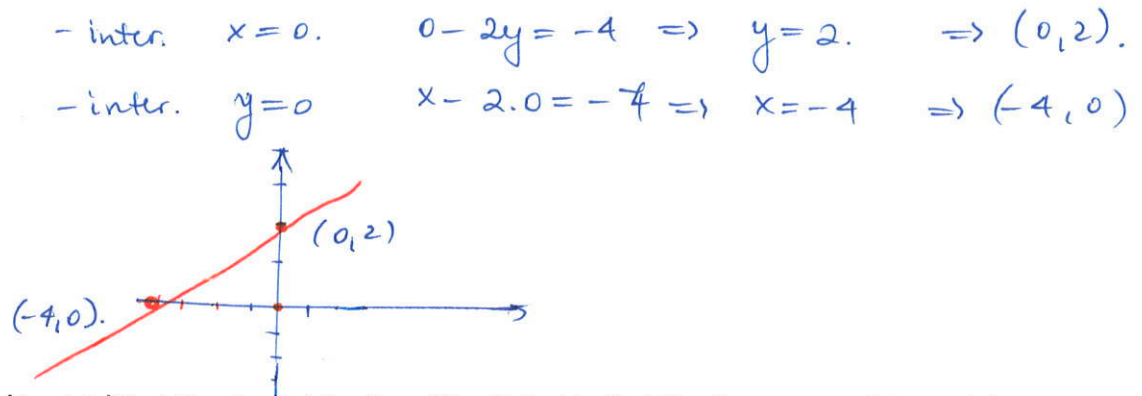$$\frac{5x + 2}{3x^2 - 3x}.$$

$$\frac{5x + 2}{3x(x-1)} = $$

The domain is . the set of all $x$ s.t. the denomin. is not $0$!

$\Rightarrow 3x(x-1) \neq 0 \Rightarrow$      $x \neq 0$ & $x \neq 1$

Domain : $\mathbb{R} \setminus \{0, 1\}$. ie. all real numbers except $1$ & $0$.

**Question 5.** [21 points] (20 total points) Consider the equation $x - 2y = -4$. Recall that the solution set can be interpreted graphically as a line in the xy-plane.

**(a)** [7 points] Sketch the line described. Be sure to label the x- and y- intercepts.

– inter.   $x = 0$.          $0 - 2y = -4 \implies y = 2$.          $\implies (0,2)$.

– inter.   $y = 0$          $x - 2.0 = -4 \implies x = -4$          $\implies (-4, 0)$

$(0,2)$

$(-4,0)$.

**(b)** [7 points] Find the slope of the line. (Recall that to find the slope you need two points on the line, that is which satisfy the line equation).

$-2y = -4 - x$

$y = 2 + \dfrac{x}{2} \implies y = \dfrac{1}{2}x + 2$.

$\implies$ slope is $\dfrac{1}{2}$.

alternatively:   $(0,2)$ & $(-4,0)$
are two points on the line:

$m = \dfrac{0-2}{-4-0} = \dfrac{1}{2}$.

**(c)** [7 points] Write the equation of the line in **slope-intercept** form. (You already know the slope and the intercept!)

$y = \dfrac{1}{2}x + 2$.

$y = mx + b$

**Question 6.** [7 points] Find the center and the radius of the circle with equation
$$(x+2)^2 + y^2 + 2y + 1 = 1.$$

$$(x+2)^2 + (y+1)^2 = 1 = r^2$$

radius is 1.

center $(-2, -1)$

**Question 7.** [6 points] Answer True (T) or False (F):

T (1) $\sqrt{2}$ is a complex number.

T (2) The circle with equation $(x+2)^2 + (y-2)^2 = 1$ has center $(-2, 2)$.

F (3) A quadratic equation always has two distinct solutions. — sometime not eg. $x^2 = 0$. or $(x-1)^2 = 0$.

F (4) The slope of a line is always a positive number.

T (5) The number 5 is a polynomial expression.

F (6) The line with slope 0 is a vertical line. — Line with slope 0 is horizontal!

PLEASE SHOW YOUR WORK OR EXPLAIN YOUR REASONS

Extra paper!