Ch. 7.4, 7.6, 7.7: Complex Numbers, Polar Coordinates, Parametric equations

Johns Hopkins University

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Complex Numbers - trig form

Recall from last week:

Definition (Trigonometric form of complex numbers)

Consider the complex number $z = a + ib$. Let $r = |a + ib| = \sqrt{a^2 + b^2}$ and let α be the angle between $\langle a, b \rangle$ and the positive x-axis. Then the trigonometric form of the complex number z is

$$z = r(\cos \alpha + i \sin \alpha).$$

Complex Numbers - trig form

Example (Write the complex number in standard form)

Write the complex number $\sqrt{2}(\cos(\pi/4) + i\sin(\pi/4))$ in the form $a + ib$.

Complex Numbers - trig form

Theorem

Let $z_1 = r_1(\cos\alpha_1 + i\sin\alpha_1)$ *and* $z_2 = r_2(\cos\alpha_2 + i\sin\alpha_2)$*, then*

$$z_1 z_2 = r_1 r_2(\cos(\alpha_1 + \alpha_2) + i\sin(\alpha_1 + \alpha_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\alpha_1 - \alpha_2) + i\sin(\alpha_1 - \alpha_2))$$

Proof.

Just try to compute $z_1 z_2$ and $\frac{z_1}{z_2}$. □

Complex Numbers - trig form

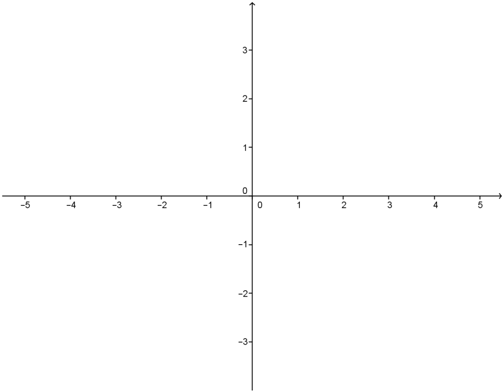
Example (Product in trig form)

Use trigonometric form to find $z_1 z_2$, if $z_1 = -2 + 2i\sqrt{3}$ and $z_2 = \sqrt{3} + i$.

Polar coordinates

Definition

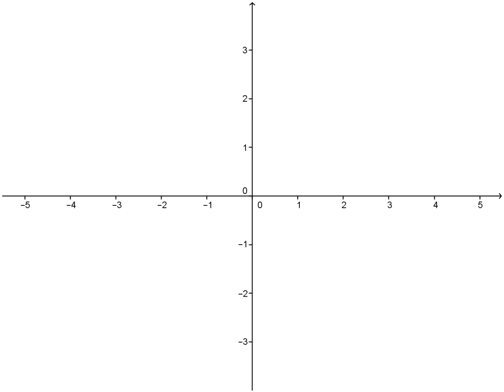
Pole, Polar axis, Polar coordinate system (directed distance and angle)



Polar coordinates

Example

Plot the points with polar coordinates $(2, 5\pi/6)$, $(-3, \pi)$, $(1, -\pi/2)$.



Polar conversion

Theorem (Conversion rules from polar to rectangular)

To convert (r, θ) *to rectangular coordinates* (x, y)*, use*

$$x = r\cos\theta$$

$$y = r\sin\theta.$$

To convert (x, y) *to polar coordinates* (r, θ)*, use*

$$r = \sqrt{x^2 + y^2}$$

and any angle θ *in standard position whose terminal side contains* (x, y)*.*

Remark

Note that we have already seen that for a vector $\vec{w} = \langle x, y\rangle$ with length r and direction angle θ, we have $\vec{w} = \langle \pm|w_x|, \pm|w_y|\rangle = \langle r\cos\theta, r\sin\theta\rangle$.

Polar conversion

Example

Convert $(6, 210^\circ)$ to rectangular.

Polar conversion

Question

What is $(\sqrt{3}/2, 1/2)$ *in polar coordinates?*

(A) $(1, \pi/6)$

(B) $(\sqrt{3}/2, \pi/3)$

(C) $(1/2, \pi/6)$

(D) $(1, \pi/3)$

Converting equations

Example

Write the polar equation as a rectangular equation.

$$r = 2\cos\theta.$$

Graphing

Example

Sketch the graph of the equation,

$$r = 2\cos\theta.$$

Hint: there are two ways - graph in the Cartesian plane or in the polar plane.

Converting equations

Example

Write the rectangular equation as a polar equation.

$$y = 3x - 2.$$

Parametric equations

Definition (Parametric equation)

An equation where x and y are both given in terms of a parameter t, that is, are functions of t.

Example (Line)

$x = 3t - 2$, $y = t + 1$, and t in the interval $[0, 3]$.

Parametric equations - graphing

Strategy: give values to the parameter to obtain values for x and y, then plot the points (x, y).

Example (Line)

Graph the parametric equations for t in the interval $[0, 3]$ and $x = 3t - 2$, $y = t + 1$.

Eliminating the parameter

We can (sometimes) eliminate the parameter and rewrite the parametric equations as one equation involving only x and y.

Example

Eliminate the parameter and then sketch the graph of the parametric equations. Determine the domain and the range.

$$x = 3t - 2$$

$$y = t + 1$$

and t in the interval $(-\infty, \infty)$.

Friday attendance

Friday is the last class meeting before the break, and we need to start a new (important) topic.

Question

Do you plan to be in class on Friday?

(A) *yes, definitely*

(B) *I would like to be, unless I oversleep or something*

(C) *no, I am travelling early*

(D) *no, because I want to sleep late / don't want to be in class / something else*

(E) *don't know yet*