Ch. 8.1: Systems of linear equations in two variables

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Linear equations

Definition (Linear equation in two variables)

A **linear equation in two variables** is an equation of the form $Ax + By = C$, where A and B are both non-zero.

Question

What is(are) the solution(s) of such an equation?

(A) *infinitely many points*

(B) *no solution*

(C) *has a solution only if A or B is 0.*

(D) *we can't say if we don't know A, B and C.*

System of linear equations

Definition (System of linear equations)

A **system of linear equations** is a collection of tow or more linear equations.

Example

$$x + 2y = 6$$

$$2x - y = -8.$$

Definition (Solution set of system of linear equations)

Solution set of a system of linear equations is is the set of ordered pairs (x, y) that satisfy all equations in the system.

Solving a system of linear equations - geometrically (1)

Example

Find the solutions of the system

$$x + 2y = 6.$$

Solving a system of linear equations - geometrically (2)

Example

Find the solutions of the system

$$x + 2y = 6$$

$$2x - y = -8.$$

Solving a system of linear equations - geometrically (3)

Example

Find the solutions of the system

$$3x - y = 2$$

$$2y - 6x = -4.$$

Solving a system of linear equations - geometrically (4)

Example

Find the solutions of the system

$$y = \frac{1}{2}x + 2$$

$$x - 2y = 4.$$

Types of systems

Definition

- Consistent system - has at least one solution
- Inconsistent system - doesn't have solutions
- Independent system - a consistent system with one solution
- Dependent system - a consistent system with infinitely many solutions

Can we tell these from the graphs?

Solving systems

There are two ways to solve systems of equations

1. substitution - expressing one of the variables in terms of the other and plugging in the second equation (very much like eliminating the parameter).
2. addition - add a multiple of one equation to the other to eliminate a variable.

Solving systems - Examples - Solve by substitution

Example

$$3x - y = 6$$

$$6x + 5y = -23$$

Example

$$3x - y = 9$$

$$2y - 6x = 7$$

Example

$$\frac{1}{2}x - \frac{2}{3}y = -2$$

$$4y = 3x + 12$$

Solving systems - Examples - Solve by addition

Example

$$3x - y = 9$$

$$2x + y = 1$$

Example

$$0.2x - 0.4y = 0.5$$

$$x - 2y = 1.3$$

Example

$$\frac{1}{2}x - \frac{2}{3}y = -2$$

$$4y = 3x + 12$$

Opinion poll

Question

What do you need most review on - from the first part of the class?

(A) *prerequisite chapter - working with expressions with exponents*

(B) *sketching polynomials*

(C) *inverse functions*

(D) *polynomials equations*

(E) *polynomial inequalities*

Opinion poll

Question

What do you need most review on - from the second part of the class?

(A) *logarithms*

(B) *trigonometry*

(C) *vectors and complex numbers*

(D) *polar and parametric equations*

(E) *systems of linear equations*

Opinion poll

Question

What do you need most review on - trigonometry?

(A) *showing something is a trig identity*

(B) *solving trig equations*

(C) *sketching trig functions*

(D) *solving triangles*

Remark

Let me know if I missed anything!!