Chapter 2

(2.1).

Def: A function is a rule that assigns to each element in one set to a unique element in a second set.

Equivalently: a function is a set of ordered pairs in which no two ordered pairs have the same first coordinate & different second coordinates.

$f(x)$ = funct. in one variable - x. (funct. notation).

Ex: (We have already seen functions!)

$f(x) = x^2 + x + 1.$ ← polynomial functions.

$f(x) = 2x+1$ ← linear functions (have lines as graphs).

To think of these as ordered pairs set $f(x)=y$.

Then the pair (x, y) will be a point on the graph of the function. Ex: for $f(x)=2x+1 \Rightarrow x=1 \to y=3 \Rightarrow (1,3)$ is on the graph of f.

(1) Note that $y=f(x)$ depends on x. (ie. depends what we plug in for x. We call y the dependent variable & x the independent variable.

(2) Note: The condition that to every element we assign a unique element in the second set means that if we plug in one value for x we will always get the same value for y:

Ex: $f(x) = y = 2x+1$ for $x=1 \Rightarrow y=3$ (always)

(we can't get $y=4$ (for example) when $x=1$).

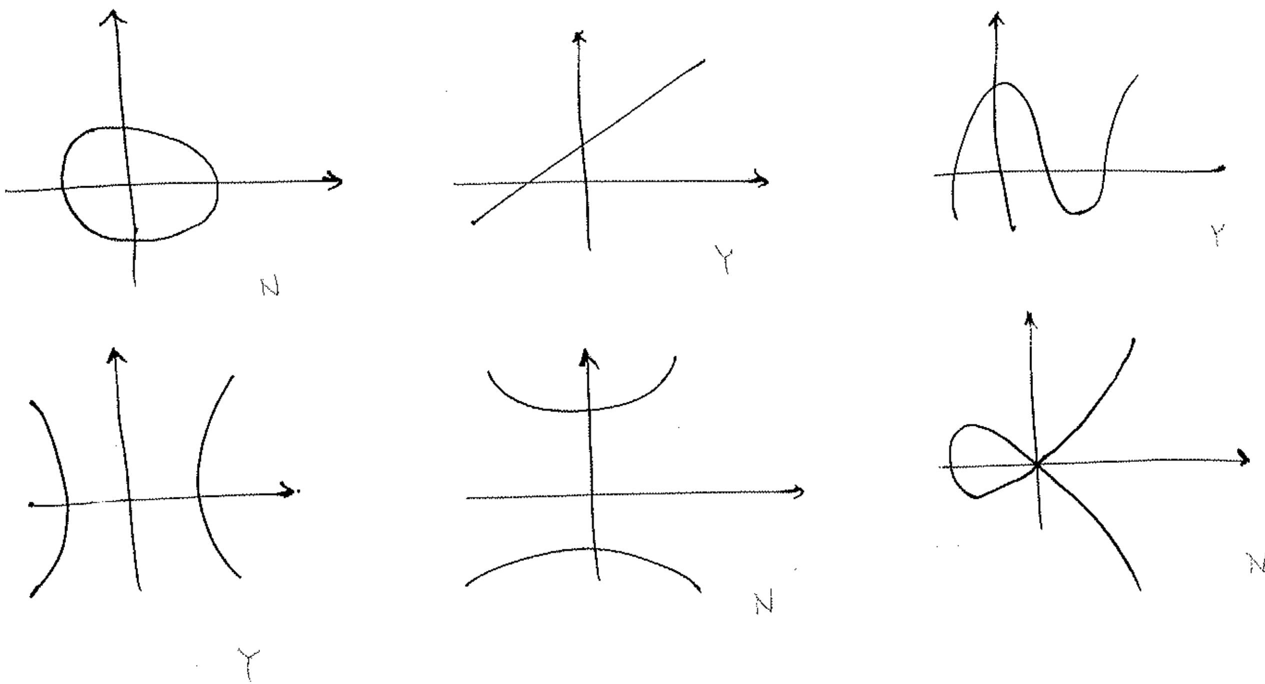
or $y=3$

We can use this to identify the graphs of functions!

Thm: A graph is the graph of a function if and only if there is no vertical line that crosses the line more than once.

(recall points on vertical lines have the same x-coordinate but different y-coordinate).

Ex:



Which of the above are functions of x?

Ex: Which of the equations define y as a function of x?
(Recall: for the same value of x we want to get only one possible value for $y = f(x)$!)

1) $x = |y|$

for $x = 1 \Rightarrow$

we get $|y| = 1$

but then $y = \pm 1$

$\Rightarrow y$ not a funct. of x

2) $y = x^2 + x + 3$

for any x we get just one y.

y is a funct. of x.

3) $x^2 + y^2 = 1$ eq. of a circle!

$y^2 = 1 - x^2$

$y = \pm\sqrt{1 - x^2}$

for $x = 0$

$y = \pm\sqrt{1} = \pm 1$

$\Rightarrow$ does not def. y as a funct of x.

Domain & Range of a function: (we have seen domains when we studied rational expressions).

Def: Domain is the set of all values/numbers that can be input of the function. (all possible first coordinates).

Range is the set of all possible second coordinates all values ~~to~~ tha the function can attain.

Note: For us both domain and range are subsets of real numbers ($\mathbb{R}$).

Ex: $f(x) = \sqrt{3-x}$

the domain are all real numbers such that $3-x \geq 0$

that is $x \leq 3$ ie. $(-\infty, 3)$.

the range are all y st. $y \geq 0$. (because the $\sqrt{\ldots}$ is always ≥ 0).

§2.2. Graphing functions. (Find domain & range for all).

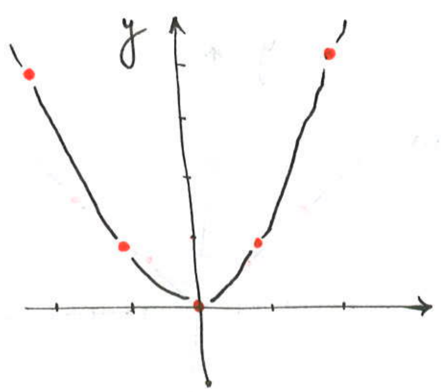
(some general funct's we need to know how to graph & what the graph looks like we will use these a lot!).

① $y = x^2$ (parabola)

Make a table for the values:

x	-2	-1	0	1	2
$y=f(x)$	4	1	0	1	4

Plot the values & connect

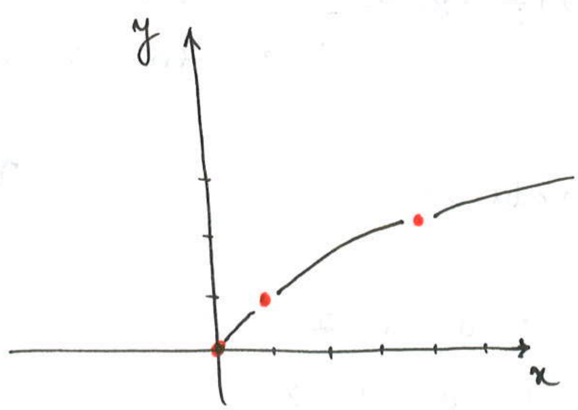


② $y = \sqrt{x}$

Table of values:

x	0	1	4	9
y	0	1	2	3

Plot & connect

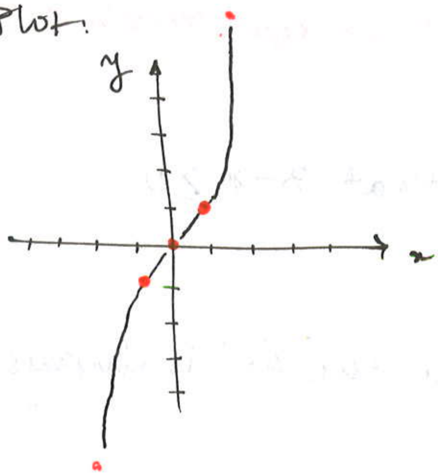


③ $y = x^3$

Table with values:

x	-2	-1	0	1	2
y	-8	-1	0	1	8

Plot:

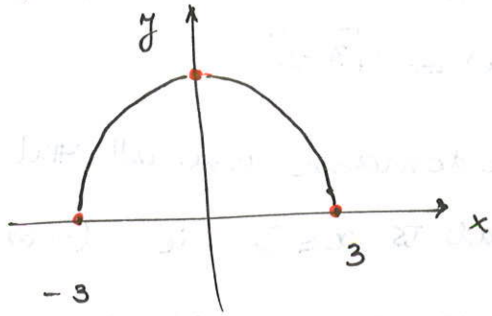


④ $y = \sqrt{a^2 - x^2}$ semi-circle a = radius

(take $a = 3$) domain?

x	-3	-2	-1	0	1	2	3
y	0	$\sqrt{5}$	$\sqrt{8}$	3	$\sqrt{8}$	$\sqrt{5}$	0

Plot:



Q: why are we not trying to graph a circle?
(is it a function of x?)

Piecewise functions

$$f(x) = \begin{cases} f_1(x) & \text{for } x \in (a,b) \\ f_2(x) & \text{for } x \in (b,c) \\ \vdots \end{cases}$$

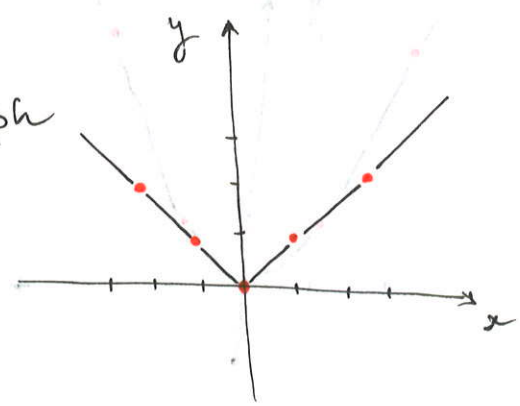
for different intervals of the real line it is a different function.

Ex: $f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0. \end{cases}$

table of values:

x	-2	-1	0	1	2
y	2	1	0	1	2

the graph



Increasing, Decreasing, Constant functions:

Def: A function is <u>increasing</u> if for $a<b$, $f(a)<f(b)$ (for a & b in the domain).

A function is <u>decreasing</u> if for $a<b$, $f(a)>f(b)$. (for a & b in the domain).

A funct. is <u>constant</u> if for any a & b in the domain $f(a)=f(b)$.

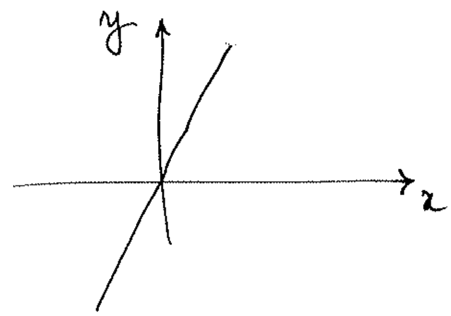
Ex: $f(x)=2x$

for $x=1<2=x$.

$f(1)<f(2)$

$\Rightarrow$ funct. is increasing.

this can be seen on the graph too:

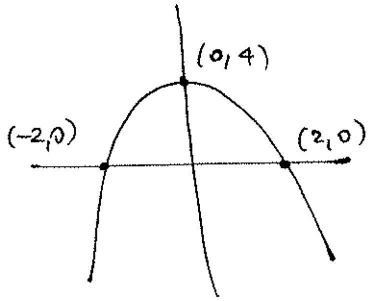


* Constant functions have horizontal graphs. (& 0 slope)

Note:

A function does not need to be only increasing, only decreasing or only constant.

Ex:



$f(x)=4-x^2$

f is increasing on $(-\infty,0)$
& decreasing on $(0,\infty)$.

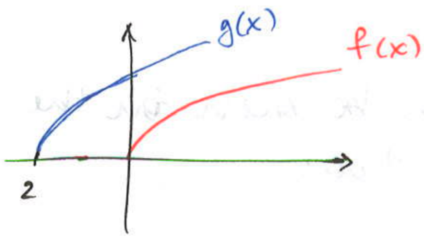
§2.3 More graphing

We already know the graphs of some functions, we can easily now obtain (by transforming those) the graphs of other functions.

① Translation (horiz.) i.e. Obtain $y = f(x \pm h)$ from $y = f(x)$.

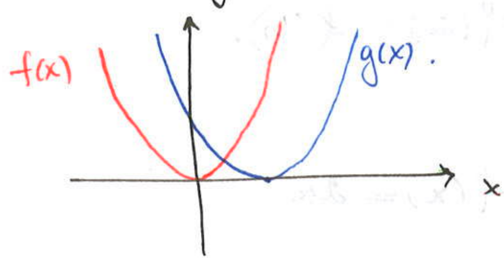
Ex: $f(x) = \sqrt{x}$

$g(x) = \sqrt{x+2} = f(x+2)$



Ex $f(x) = x^2$

$g(x) = (x-2)^2 = f(x-2)$.

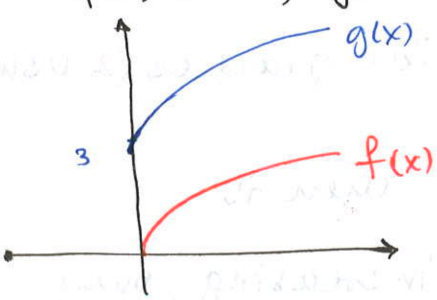


② Translation (vertical) i.e. obtain $y = f(x) + k$ from $y = f(x)$.

if $k \geq 0$ translation by k units upwards

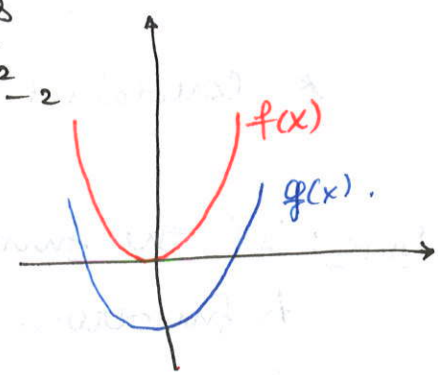
if $k < 0$ translation by k units downwards

$f(x) = \sqrt{x}$, $g(x) = \sqrt{x} + 3$



$f(x) = x^2$

$g(x) = x^2 - 2$



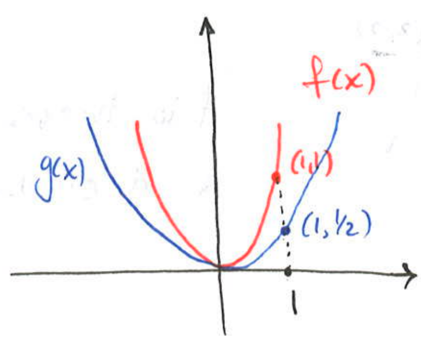
③ Scaling (shrinking & stretching) i.e. obtain $y = a f(x)$ from $y = f(x)$

if $a > 1$ – stretching

if $0 < a < 1$ – shrinking.

$f(x) = x^2$

$g(x) = \frac{1}{2}x^2$



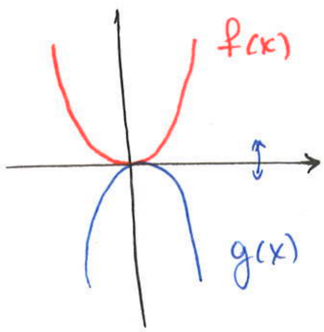
for $a < 0$ see reflection!

④ Reflection, obtain $y = -f(x)$ from $y = f(x)$ (wrt x-axis)

(this is like scaling with $a = -1$).

Ex. $f(x) = x^2$

$g(x) = -x^2$



Symmetry when a reflection occurs by the graph remains the same.

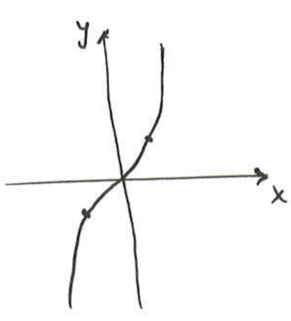
Ex: the graph above $f(x) = x^2$ is symm. with respect to (wrt) the y-axis.

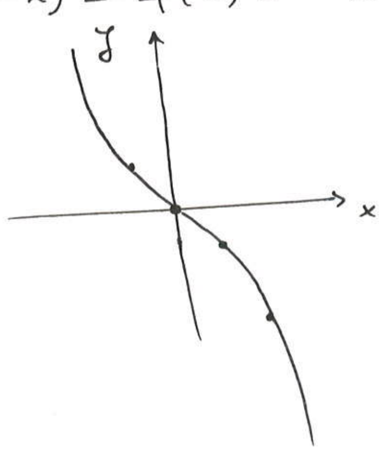
Def! **Even functions** are functions that are symmetric wrt about the y-axis

In particular $f(x) = f(-x)$.

Def: **Odd functions** are functions that are symmetric about the origin: In particular $f(-x) = -f(x)$.

Ex: $f(x) = x^3$ $\qquad$ $f(-x) = -f(x) = -x^3$





x	-1	0	1	2
y	-1	0	1	8

x	-1	0	1	2
y	1	0	-1	-8