Worksheet for the Final

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This is the sheet with practice problems. We will work on them in the last two classes. You can decide which problems to work on, my advice is - chose the ones from the chapters which you don't feel comfortable with. You don't have to solve all of them. Please note that some of these problems might be harder than the ones you can get on the final exam but I believe that hard problems can teach you a lot.

1. Simplify

   (a)
   $$\left(\frac{27}{8}\right)^{-\frac{2}{3}}$$

   (b)
   $$\left(\frac{\frac{1}{x^2}}{y^5}\right)^{\frac{1}{2}}$$

   (c)
   $$\left(\frac{\frac{a^{-1/2}}{\sqrt{b}}}{\frac{3}{a}}\right)^2$$

2. Factor the following polynomials

   (a)
   $$x^4 - 2x^3 - 4x^2 + 10x - 5$$

   (b)
   $$x^3 + 5x^2 + 6x$$

   (c)
   $$x^3 - 2x^2 + x - 2$$

3. Find the inverse of the following functions and sketch them.

   (a)
   $$\frac{x-2}{x-3}$$

(b)

$$\log_2(x)$$

(c)

$$\log_2(x-1)$$

(d)

$$e^x + 1$$

4. Let $f(x) = \sqrt{x+1}$ and $g(x) = e^x - 1$
   (a) Compute $f + g$, and $f \circ g$
   (b) What is the domain of $f + g$, and $f \circ g$
5. Let $f(x) = \tan x$ and $g(x) = 2x$
   (a) Compute $f + g$, and $f \circ g$
   (b) What is the domain of $f + g$, and $f \circ g$
6. Find the domain of the following functions
   (a)

$$\frac{x-2}{(x-1)(x+3)}$$

   (b)

$$\frac{x-2}{\log x}$$

   (c)

$$\frac{1}{e^x}$$

   (d)

$$\frac{x+1}{e^x - 1}$$

   (e)

$$\frac{\ln x}{x-1}$$

   (f)

$$\frac{1}{\sin x}$$

   Hint: Remember we have discussed what the domain of $\frac{f(x)}{g(x)}$ is. Be cautious, some of these are tricky.
7. Find the solutions of the following inequalities
   (a)

$$-3x + 1 \leq 0$$

(b)
$$2x + 2 \geq 0$$

(c)
$$(x-3)^2(x+2)x \leq 0$$

(d)
$$(x-3)(x+5)(x+2)x^2 > 0$$

(e)
$$\frac{x-3}{(x+2)(x-1)^2} \geq 0$$

(f)
$$\frac{(x-3)^2(x-1)}{(x+2)(x-1)^2} \leq 0$$

8. Graph the following functions

(a)
$$(x-3)(x+5)(x+2)x^2 = 0$$

(b)
$$\log_2(x-1) + 2$$

(c)
$$2\tan(3x)$$

9. Evaluate/Solve

(a)
$$e^{3\ln 2}$$

(b)
$$\ln(x+1) = 3$$

(c)
$$2^x = 4^{x-1}$$

(d)
$$\log(x-1) = \log(x) + \log(2x-1)$$

(e)
$$\ln(e^{\ln x})$$

(f)
$$\log_x(4) = 2$$

10. Solve the triangle (if it exists)

(a) Solve the SSS triangle with sides a=3, b=4 and c=5.

(b) Solve the SAS triangle with $a = 1$, $b = 2$ and $\gamma = 15°$

(c) Solve the ASA triangle with $a = 1$, $\beta = 35°$ and $\gamma = 15°$

(d) Solve the SSA triangle with $a = 2$, $b = 4$ and $\alpha = 30°$

11. Solve the trig equations:

(a) Find all x such that

$$\cos(x) = \frac{\sqrt{3}}{2}$$

(b) Find $\tan(x)$ if $\sin(x) = \frac{1}{2}$ and x is in the second quadrant.

(c) Solve $\arctan(0) = x$.

(d) Solve $\tan(x) = -\sqrt{3}$.

12. Show the following trig identities

(a)

$$\cos(-2x) - 2\cos^2(x) + 1 = 0$$

(b)

$$\tan^3(x) + \tan(x) = \sec^3(x)\sin(x)$$

(c)

$$\cos^4(x) - \sin^4(x) = \cos(2x)$$

13. Let $\vec{A} = \langle 1, 2\rangle$ and $\vec{B} = \langle -9, 6\rangle$ be two vectors. Compute both algebraically and geometrically:

(a)

$$2\vec{A} - \frac{1}{3}\vec{B}$$

(b) Find the magnitude of

i.

$$\vec{A}$$

ii.

$$-2\vec{A}$$

iii.

$$\frac{1}{2}\vec{A}$$

(c) Write $\vec{A}$'s initial and terminal point in polar coordinates

(d) What's is the directions angle of $\vec{A}$? And of $\vec{B}$

(e) What is the dot product of $\vec{A}$ and $\vec{B}$.

(f) What is the angle (or at least cos of the angle) between $\vec{A}$ and $\vec{B}$?

(g) Is $\vec{A}$ a unit vector? Is $\vec{B}$ a unit vector?

14. Consider the complex number $z = \sqrt{3} + i$,

(a) Find the trig form of z.

(b) Use the trig form to calculate z^2.

(c) What's the absolute value of z?

15. Polar coordinates

(a) Write the point $(\sqrt{3}, -1)$ in polar coordinates.

(b) Write the point $(3, 210^\circ)$ in rectangular coordinates.

16. Sketch the parametric equations

(a) for $t \in \mathbb{R}$

$$x = 3t + 1$$
$$y = t - 1$$

(b) Eliminate the parameter and sketch the new equations to check your answer from the previous question.

(c) for $t \in [0, 2\pi]$

$$x = \cos x$$
$$y = \sin x$$

(d) Try to write the equation in Cartesian (rectangular) coordinates. Use the fact that you know what the graph looks like.

17. Solve the following systems of equations both algebraically and geometrically. Determine of the systems are consistent/inconsistent, dependent/independent

(a) Solve

$$\tfrac{1}{2}x - \tfrac{2}{3}y = -2$$
$$4y = 3x + 12$$

(b) Solve

$$3x - y = 6$$
$$6x + 5y = -3$$