Review problems for the final

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1. Find the limit without using L'Hopital

   (a)
   $$\lim_{x\to\infty} \frac{e^{2x} + x^2}{e^{3x}}$$

   (b)
   $$\lim_{x\to 0} \frac{\tan(x)}{x}$$

2. Determine where the function is continuous, and where it is differentiable:

   (a)
   $$f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x - 2} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

   (b)
   $$g(x) = \ln \frac{x + 1}{x - 1}$$

3. Differentiate:

   (a)
   $$f(x) = 3x^2 + 5x - 1$$

   (b)
   $$g(x) = \sin(x) + e^x$$

4. Differentiate:

   (a)
   $$y = \ln(\cos(x))$$

   (b)
   $$y = (2^x)(x^2)$$

5. Estimate:

   (a)
   $$\sqrt{45}$$

(b)
$$\cos(0.1)$$

6. Sketch the graph (find asymptotes, critical points, inflection points, local extrema, etc)

(a)
$$e^x - x^2$$

(b)
$$\frac{x^3 - 8}{x^2 + 4}$$

7. Find the limit:

(a)
$$\lim_{x \to \infty} (x + 2)^x$$

(b)
$$\lim_{x \to \infty} \frac{\ln(x + 2)}{x}$$

Bonus:
$$\lim_{x \to 0} \frac{\ln(1 - x) - \sin(x)}{1 - \cos^2(x)}$$

8. Find the area

(a)
$$\int_{1/2}^{1} \sqrt{1 - x^2}\, dx$$

(b)
$$\int_{-3}^{0} (4 - \sqrt{9 - x^2})\, dx$$

9. Without computing the integral show

(a)
$$\int_{1}^{2} x\, dx \leq \int_{1}^{2} x^2\, dx$$

(b)
$$\pi/3 \leq \int_{\pi/6}^{5\pi/6} \sin x\, dx \leq 2\pi/3$$

10. Find dy/dx

(a)
$$y = \int_{x^3}^{x^4} \ln(1 + t^2)\, dt, \quad for \ x > 0$$

2

(b)
$$y = \int_{-2x}^{x^2} (1 + te^t) dt$$

11. Compute

(a)
$$\int x^2 \sqrt{x} dx$$

(b)
$$\int \frac{(3x + x^3)}{2\sqrt{x}} dx$$

(c)
$$\int_0^{\pi/2} sin(3x) dx$$

(d)
$$\int \frac{t}{t+2} dt$$

12. Suppose that
$$\int_0^x f(t) dt = e^{x^2},$$

Find $f(x)$.

13. Find the volume of the solids obtained by rotating the region, bounded by the given curves about the x-axis.

(a)
$$y = 2 - x^3, \ y = 2 + x^3, \ 0 \le x \le 1.$$

(b)
$$y = x^2, \ y = x, \ 0 \le x \le 1.$$

14. Find the volume of the solids obtained by rotating the region, bounded by the given curves about the y-axis.

(a)
$$y = \ln(x + 1), \ y = \ln 3, \ x = 0$$

(b)
$$y = x^2, \ y = \sqrt{x}, \ 0 \le x \le 1.$$

15. Compute the integrals

(a)
$$\int xe^{-x^2/2} dx$$

(b)
$$\int \cos(2x - 1)dx$$

(c)
$$\int \frac{(\ln x)^2}{x}dx$$

(d)
$$\int \frac{\sin x}{\cos^2 x}dx$$

16. Compute the integrals

(a)
$$\int_0^1 xe^{-x}dx$$

(b)
$$\int_1^4 \ln(\sqrt{x})dx$$

(c)
$$\int e^x \sin x dx$$

(d)
$$\int e^{\sqrt{x+1}}dx$$

(e)
$$\int \frac{1}{\tan x}dx$$

17. Compute the integrals

(a)
$$\int \frac{x^2 + 1}{x^2 + 3x + 2}$$

(b)
$$\int_0^1 \frac{x}{x^2 + 1}$$

(c)
$$\int \frac{x^2 + 4}{x^2 - 4}$$

(d)
$$\int \frac{2x^2 - 3x + 2}{(x^2 + 1)^2}$$