Question 3.6.12

We need to prove that $\lim_{x\to 1} 1/x = 1$. Let $f(x) = 1/x$, then we need to show that for every $\epsilon > 0$ there exists a δ such that $|f(x) - 1| < \epsilon$ whenever $|x - 1| < \delta$.

Now we have

$$|f(x) - 1| < \epsilon \iff |1/x - 1| < \epsilon \iff |\frac{x-1}{x}| < \epsilon \iff |x-1|\frac{1}{|x|} < \epsilon.$$

On the other hand we have $|x-1| < \delta$. I can assume $\delta \leq 1/2$. This is a valid assumption to make since, in general, once we find a δ that works, all smaller values of δ also work.

Now we obtain that

$$|x-1| < 1/2 \iff -1/2 < x - 1 < 1/2 \iff 1/2 < x < 3/2.$$

hence for the reciprocals we have $2 > 1/x > 2/3$, and in particular $2 > 1/x$ and $2 > |1/x|$. So now we can use this to get:

$$|x-1|\frac{1}{|x|} < 2|x-1| < \epsilon$$

and so $|x - 1| < \epsilon/2$ so we can take $\delta = \epsilon/2$.